METRO MANUAL

BAUSCH & LOMB OPTICAL CO.
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A. E. Howe
1917

METRO MANUAL,

A [illegible] Engineers,

Containing [illegible] The

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Bausch [illegible] Optical Co.

Rochester, N. Y.

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Executive Office and Manufactory of the Bausch & Lomb Optical Co., Rochester, N. Y.

METRO MANUAL,

A Hand Book for Engineers

Containing Technical Information Regarding The Construction, Adjustment and Use of

Transits, Tachymeters, Theodolites, Alidades, Levels, Etc.

Manufactured by

Bausch & Lomb Optical Co.

Rochester, N. Y.

Branches:

New York: 200 Fifth Ave. **Washington:** 613 Fifteenth St., N. W.

Chicago: 122 S. Michigan Blvd. **San Francisco:** 154 Sutter St.

Frankfurt a. M. G. m. b. H. 31 Schillerstrasse.

London: 37 & 38, Hatton Garden, E. C.

"Here we watch the race grappling with great intellectual difficulties and we see the spectacle of her champions painfully but surely overcoming countless obstacles. Each of their victories is a genuine victory of the spirit, each of their defeats a spiritual chastisement.

* * *

"It is not the *mind* but the *method* the race has advanced; and where we are superior to our forefathers is in the fact that we have learned—at any rate in science—first to lay a solid foundation of fact before we begin to theorize."—*R. C. Maclaurin.*

PREFACE.

. Efficiency, as a distinct branch of contemporaneous activity, has its origin in the complexity of modern life, in the diminishing margin of natural resource and the relentless crusade against time wasted in unprofitable results. It demands honest effort, intelligent interpretation and a replacement of tradition with obviously better methods.

Greater refinement of method characterizes not only the larger engineering projects of to-day but dominates policy in the manufacture of the better grades of equipment. All logical enterprises meeting a natural demand have an upward tendency—a constant trend toward progress and evolution.

In the manufacture of Surveying Instruments, the problem of producing parts to pre-determined standards of accuracy is a matter of systematic insistence. Inferiority is the inevitable response to a demand for quantity in preference to quality, and *precision* instruments have never been manufactured upon the basis of lowest unit cost or maximum output.

All engineering work is founded upon an effort to utilize power, materials and equipment without preventable waste and to bring actual performance up to the level of an accepted standard of comparison. Such investigations require a critical study of the nature and source of error as well as logical conclusions as to their relative importance.

In linear measurement the origin and magnitude of errors are reasonably tangible quantities; but in angular measurement the effect of natural, instrumental and personal influences tends to modify accuracy. On the rational and commonly accepted assumption that the absolute value of an angle is never known, we can corollate the proposition that instrumental errors can be fairly compensated by the systematic process of reversion, repetition and equalization; but where the result is affected by personal inefficiency there is no ground for discussion .

On the other hand, the more confidence an engineer reposes in his own capabilities, the more critical and exacting he becomes concerning each inscrutable detail of construction for which the maker alone is responsible. The instinctive tendency in every high class man is toward higher class results, partially accomplished at least through the elimination of time or patience lost in unstable instruments.

To extend the boundaries of popular knowledge on this subject, to provide a ready reference in relation to instruments of our own manufacture and to give our customers the best there is in value and treatment without their insistence, is the excuse and necessity for the publication of this, the ninth, enlarged and revised edition of the Saegmuller Vest Pocket Handbook.

B. & L. O. CO.

Rochester, N. Y.
1915

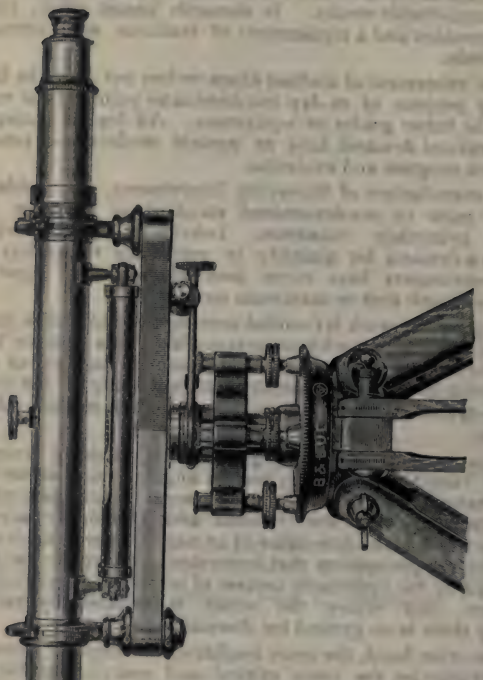

Fig. 1—Bausch & Lomb Wye Level. Made in 16-inch and 21-inch models. Interior focus; perfect balance.

THE WYE LEVEL

THE Y-Level was invented by Jonathan Sissons of London in 1740. The general design has never been changed, for revision of details which make for improvement is necessarily restricted by the fundamental principles of construction. Our latest improvements include a new method of interior focusing, which reduces the possibility of derangements in collimation, and a torsionless mounting for the level vial, which prevents undue strain in varying temperature.

The routine of adjustment is comparatively simple, but permanency and stability are so dependent upon mechanical conditions in the collars that, for the more particular work, adjustments should be tested and rectified frequently. The prime accomplishment sought in the adjustment of any leveling instrument, of whatever design, is parallelism between the bubble axis and the sight-line. All details of construction and all methods of adjustment are only a means to this end, for, by its attainment, the sight-line can be relied upon as lying in the true horizon when the bubble occupies the center of its scale.

The vertical axis should be regarded only as means by which the sight-line, so adjusted, may be directed into any position of the field; but for convenience sake it is highly desirable that the vertical axis should be perpendicular to the sight-line and that it should be otherwise provided with those means by which it can be adjusted to perfect verticality.

The collars determine the geometrical axis of the tube and are at the basis of the principal adjustments. The sight-line is collimated, or the cross hairs are brought into the optical axis, by longitudinal revolution in the collars, on the one hand, and the bubble-axis is adjusted to the same collar-axis by end-for-end reversals, on the other. On the assumption that things which are equal to the same thing are equal to each other, we must feel satisfied that the collimated line of sight will be parallel to the bubble-axis.

This is a safe speculation in a new instrument which is also free from eccentricity of mounting in the objective; but when the collars become either conical or eccentric through continued wear, this method of comparison should not be considered as infallible. In such a case the peg method, with a test base at least 100 feet each side of the center, is the only field method upon which the engineer can place dependence. (See page 20.)

In the process of adjustment six elements of construction should be considered. These include:—

1. *The line of sight*, which passes through the intersection of the cross wires, whatever their position in the tube, and the nodal point in the objective.

2. *The line of collimation*, which is the final position of the sight line when adjusted to coincidence with the optical axis of the objective.

3. *The collar axis*, which is the geometrical axis of the tube.

4. *The bubble axis*, which is a line, tangent to the curved surface of the vial at the center of its scale.

5. *The wye axes*, which are elements in the imaginary cylinder determined by the bearing points, or pivots. Its position is determined by the relative height of the wyes above the base bar.

6. *The vertical axis* of revolution, which passes through the spindle.

The Collimation Adjustment is intended to bring the horizontal cross-wire into the equator of the field of view or, as noted above, to arrange the line of sight so that it will pass normally through the optical axis of the objective.

To perform this operation it is not necessary that the telescope shall be perfectly horizontal. The test could be conducted in two V-notches cut in a cigar box, or by any other such expedient.

Set up the instrument firmly, open the clips, sight some distant object, and suppose, as in Fig. 2, the original position of the line of sight to be indicated by the finely dotted lines. Revolve the telescope carefully in the wyes half way round and let the second position of the cross wires be indicated by the heavier dotted lines. In effect, the tropics have been located and the equatorial line, mid-way between them, will be the position sought.

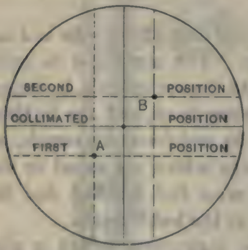

Fig. 2

Move the horizontal hair into the equator of the field by turning the upper and lower diaphragm studs in the same direction, first by slightly loosening the one, then by tightening the other. When the correct position has been secured, the horizontal wire will continue to cut the same point when the telescope is revolved on its own axis.

In the Bausch & Lomb instrument, the diaphragm adjusting studs are concealed beneath a ferrule which forms a guard against accident or tampering as well as an additional protection against moisture. Remove the ferrule to adjust the diaphragm as suggested. If the eyepiece is of the erecting type, move the diaphragm in the direction opposite the one which seems to be necessary; but if the ocular is of the inverting type, it will give a positive image of the field and the diaphragm may be moved in the direction apparently necessary.

The supreme test of the **accuracy of the draw-tube** and the capacity of the instrument to perform accurate work at all distances in the field, now requires that the collimation adjustment shall be repeated on a point not over 12 to 15 feet from the instrument. This necessitates racking out the focusing system, by which any lateral displacement will be plainly manifest on inverting the telescope. When a condition of this kind exists, the mean of double observations should be taken or the matter should be reported to the manufacturer.

In the example suggested in Figure 2, a side adjustment for the vertical wire should be secured mid-way between the two outside positions; but it is not so important in a level as the proper location of the horizontal wire. A level would work successfully without a vertical wire, but it is always convenient for reference in plumbing the rod.

It is very desirable that the horizontal wire shall be truly horizontal over the entire span of the field. When all the adjustments are completed, as accomplished by the following directions, and the vertical axis adjusted to perfect verticality, close the clips and revolve the instrument slightly from side to side while sighting on a test point. If the point appears to rise above the horizontal wire at one end and drop below at the other, all the studs of the diaphragm should be loosened and one of them tapped gently in the direction desired. This process may slightly disturb the adjustment for collimation, and in this event the collimation test should be repeated.

If the **Eyepiece** is not perfectly centered, the collimated cross wires may not appear to be in the exact center of the field of view, but this state of affairs need cause no apprehension. If the cross wires and the field of view appear to move around in a small circle, eccentricity in mounting is evident; but if the cross adheres to the test point on inversion, the utmost requirements in the case have been fulfilled and the further rectification can only be accomplished by aligning the axis of the eyepiece with that of the objective.

Eccentricity in the Eyepiece will not effect accuracy of results but when the optical axis is not coincident with the collar-axis, discrepancies will occur as explained later. Our method of centering the eyepiece and dispensing with the objective draw tube does away with the confusion of three or four sets of adjusting nuts protruding from the telescope barrel, and maintains a more perfect equilibrium while focusing on different points in the field of view.*

The Bubble Adjustment proposes to secure parallelism between its tangential axis and the axis of the collars. Bring the bubble to the center of its run over either set of leveling screws and carefully reverse the telescope end-for-end, in the wyes. Correct half the error with the leveling screws and the other half with the adjusting nuts on the bubble bracket. Restore the telescope to its original position and note the location of the bubble. If the test has not been carefully made or rectified, check the operation, as directed above, over both sets of leveling screws.

This process will secure parallelism between the axis of the bubble and the contact points of the collars, quite irrespective of the relative diameters of the collars themselves. When the collars become unequal in diameter through wear, they are said to be conical and it is not impossible that either collar may also become elliptical in section through a slight variation in the density of the metal employed.

Such a state of affairs cannot help but exert an insidious influence, not only on the collimation test at short distances, but upon bubble adjustment as well. In the attempt to reconcile the bubble axis with the optical axis through a variable medium of comparison many serious and expensive errors have occurred.* *

The Lateral Adjustment, for the bubble, or the "wind adjustment" as it is sometimes called, is intended to bring the longitudinal axis of the level vial vertically beneath the axis of the tube.

Revolve the telescope in the wyes so that the vial tube will move in a short arc either side of the vertical plane. If the bubble moves out of the center, correct the entire error with the lateral adjusting screws in the lug at one end of the tube. The whole error is corrected in this case because the amount has not been determined by the process of reversion.

Repeat the test by revolving the telescope so that the level vial lies on the opposite side. If the bubble continues to run off center in the same direction as before, this will indicate that the vial is ground conically and not cylindrically. Such a discrepancy, however, is not a serious one and the side adjustment may be considered as correct, in such a case, when the bubble runs off center an equal number of divisions in the same direction at each side.

** S. P. Baird in Eng. Rec., Apr. 25, 1914, Says " * * * there will be a shift to the bubble when changing focus * * * on short sights * * * due to the extraordinary length or overhang of the telescope."*

*** Read anonymous article on "Sea Level Datum" in Ry. Eng. & M. of W., Chicago, Dec. 1914, recording a fruitless expenditure of $16,500 on 1462 mi. of precise control (?) with ordinary instruments.*

The adjustment for parallelism should now be tested again, for these adjustments are inter-related and both are dependent upon the perfection of the collars.

, The procedure thus far does not profess to test the bubble-axis for parallelism by *direct* comparison with the line of sight. The collar axis is the intermediary basis of the test. The only direct means of comparison available in the field is the "peg method", and with worn-out collars this is the only method by which the Y-level can be adjusted.

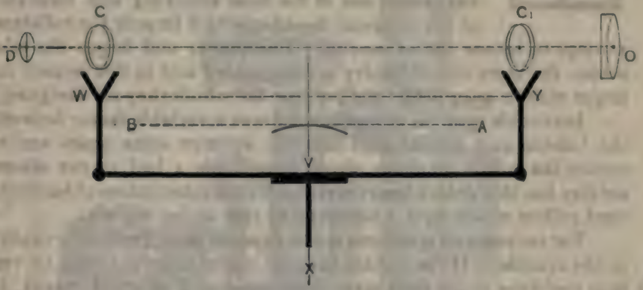

Fig. 3

Theoretically we now have the optical axis, DO, which is understood to be coincident with the collar axis, CC_1 and the bubble axis, BA, arranged in parallel lines, which for convenience of operation, must be adjusted to right angles with the vertical axis, VX.

Although very desirable this adjustment is not vital. The adjustments might be stopped at this point so far as accuracy is concerned, but it is highly desirable to facilitate the work that the sight-line shall describe a true horizon during a revolution on the vertical axis.

The Wye Axis is the axis of an imaginary cylinder, of which the contact points with the collars are longitudinal elements. It is immediately related to the vertical axis and is to be tested by leveling up carefully again over either set of leveling screws.

Revolve the instrument on the vertical axis 180° as closely as may be gauged by the eye; correct half the bubble displacement in the leveling base and the rest by raising or lowering one of the wye supports as the case may require. Make the test across the other set of leveling screws and continue the operation until the bubble remains centered during an entire revolution.

This operation brings the axis of the bubble perpendicular to the vertical axis and incidently adjusts the vertical axis to perfect verticality. It can always be perfected without respect to conditions in the collars or others portions of the instrument. The chief precaution in making the test is to look for lost motion in the vertical axis.

THE PRECISE WYE LEVEL

THE ordinary Y-level has reached the paradoxical distinction of being at once the most popular and most fallible of any topographical instrument, with the possible exception of the surveyors' compass.

The tendency to increase the length of the telescope involves heavier objective mounts and the re-distribution of weight in the process of focusing has caused one of the most annoying and incurable of the temporal disorders which impede the collateral attempt to secure better results. The longer and larger the telescope, the more this difficulty is exaggerated and in this respect the larger models only defeat the purpose for which they are designed.*

Inevitable wear in the collars induces a variable factor between the bubble and optical axes and the attempt some years ago to mount the collars on agate pivots only made a bad matter worse; neither has this defect been overcome by the substitution of hardened steel collars which have a tendency to rust under neglect.

For success and speed too much depends upon perfect verticality in the spindle. If the bubble runs off center when directed to various positions in the field there is no rapid and effectual means, in an ordinary wye with four screw base, to control the bubble at the instant of observation. Strain communicated through the comparatively coarse adjustment of the leveling screws induces secondary strains in the vertical axis that are aggrivated as the temperature changes. The consequence is that a sluggish bubble that will not indicate disturbances has gone a long way toward making the ordinary wye level "precise" and "stable in adjustments".

Investigations looking into these various sources of error led J. F. Brander of Augsberg to re-design the lower portion in 1769. † He overcame the necessity for a height adjustment in the wyes, corrected all the difficulties here discussed and secured greater rapidity of manipulation by a "Vertical Control" or elevation screw operating between two pivoted elements of the base-bar.

Brander first placed the pivots at one end of the instrument. In 1842 Ertel located them at the center of the base-bars as we have them in the model presented on the next page. Since that time no essential modification of these ideas has been introduced until the interior system of focusing was invented to reduce to a negligible minimum errors arising from imperfect balance or temporal derangement in the collimation adjustment, and the three screw base (pp. 18 & 139) which provides the most rapid known method of correcting slight bubble displacements at the instant of observation.

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* *In 1904, Mr. Roman Seelig introduced an exceptional construction that preserved a better equilibrium by moving the objective and eyepiece mounts simultaneously in opposite directions with a pinion operating in a double rack.*

† *See Pacific Builder and Engineer, Seattle, May 31, 1914, p. 337.*

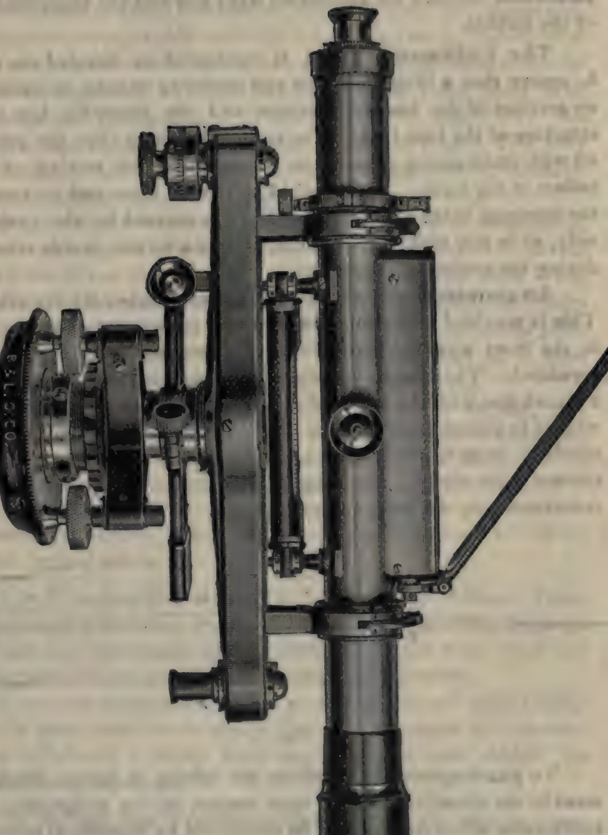

Fig. 2—Bausch & Lomb Precise Y-Level
Cryptic Focus, Supplementary Vial, Reading Mirror, etc.

The adjustments of the precise wye are more simple than the ordinary type. Nothing need be seriously considered but the parallelism between the collimated sight-line and the tangential axis of the bubble.

The Collimation Test is conducted as directed on page 6, except that a little more care and dexterity should be exercised on account of the heavier telescope and the avowedly less rigid structure of the base-bars. The automatic spring clips are provided with small spring plungers to insure a perfect seating of the collars in the wyes. These should be thrown open and in turning the telescope in the wyes it should not be grasped by the eyepiece only, or in any other way that may induce a torsion or side pressure during the process of revolution.

An annular clamp encircles the telescope tube near the ocular. This is provided with cams and adjusting screws that engage a lug in the front wye so that 180° revolutions can be very exactly regulated. They are to be adjusted by sighting the telescope on a plumb line with the lower adjusting screw set so that the vertical wire is in coincidence with the line. Revolve the telescope very carefully, so as to avoid a shock when the other set-screw comes in contact with the lug, and bring the vertical wire once more into coincidence by the other adjusting screw.

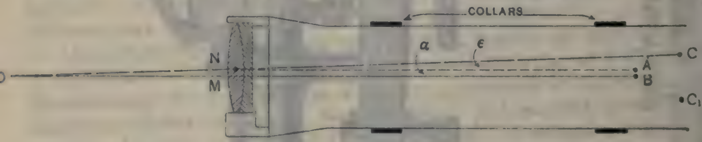

Fig. 5

To guard against, or to reduce the effects of lateral displacement in the objective mount, with respect to the collar-axis, the collimation adjustment should be conducted by sighting at infinity, or upon some object in the distant horizon. Otherwise a slight angular error will creep in, expressed by the formula:

$$\epsilon = \frac{a}{D \ \sin \ 1''}$$

in which *a* is the amount of eccentricity in the objective, above or below the geometrical axis of the collars, and D the distance to the object sighted.

In Fig. 5, let OMB represent the geometrical axis of the tube, as determined by the collars, and let it be assumed that the objective is eccentrically mounted as indicated.

If such an objective were collimated on a very distant object, the incident beam would enter the objective in nearly parallel rays and, whether it were right-side-up or up-side-down, the collimated cross wires would occur in the optical axis at the principal focus at A; the line of collimation, NA, would be parallel to the collar axis, MB, and this would still be true if, in addition to its eccentricity, the objective were slightly inclined to the axis of the tube. The only error induced, therefore, while leveling on distant objects, would be a rather imperceptible plus or minus quantity equal to a.

When focused for collimation on a comparatively near object, as at O, the image will be formed on the line joining O with the nodal point of the objective, N, as at C, where the cross wires would have to be located. If the telescope were inverted, the image would occur at C_1 but the cross wires would also occur at this point and no error would be noticed except that the field and cross wires would appear to move around together in a small circle.

For example, let it be assumed that OM = D = 50 feet, that MN = 1 mm and that angular error = ϵ. For very small angles

$$\tan \epsilon = \frac{a}{D} = \sin \epsilon.$$

Substituting these values in the formula given above, $\epsilon = 13.5''$, or .006 feet per 100 ft.

The Principal Bubble may have a sensibility of 10 sec., 5 sec., or even more if desired. It is mounted at the top of the telescope in a brass box encasement and otherwise protected from dust and sudden temperature changes by a crystal glass window.

The cover of the box contains a reading mirror* which snaps down in place when not in use. The cover is hinged on universal pivots to absorb a shock from any direction and reduce the probability of breakage.

To Adjust the Precision Bubble turn the mirror backward and off to one side; remove the retainer-plate and slide back the glass cover only enough to expose the front adjusting screw. A preliminary test may be conducted by end-for-end reversals in the wyes but it should be verified by the pegs, particularly if there is a suspicion of unequal wear of the collars. (See page 19).

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* *W. E. Whittier in Eng. & Cont. June 12, 1912 says, "The ordinary Y-level is inaccurate for precise work and * * it is not equipped with a mirror, * * the advantages of which * * cannot be too highly recommended.*

To find the value in seconds for each division of the bubble scale, run the bubble near to the objective end of the tube, reading each end of the bubble against the graduation. Take a reading on a rod, say 400 ft. from center of instrument, move the bubble toward the eye-end and note the number of divisions traversed by each end of the bubble. Take a second rod reading and get rod interval.

Let E = divisions traversed by eye-end of bubble.
" O = " " " object-end of bubble.
" R = rod interval in feet.

Then $$V'' = \frac{R}{400 \, \frac{E + O}{2} \sin 1''}$$ (Log. Sin. 1″ = 4.6855749)

Slight inequalities in the collars will be difficult of apprehension by end for-end reversals, particularly as the upper arc is not taken into account; but this influence on a collimation test, conducted at some distance, is quite a different matter. The length of base for the bubble test is limited to the distance between the collars. Assuming the angular value of the error to be the same, it is likely to be tangible in the collimation test when an ordinary bubble would not indicate it.

In a case of this kind it is always preferable to make the bubble test by the peg method as explained on page 19. The peg method does not take the collars into consideration and their inequalities are of no concern. Disparity in contour or diameter of the collars can not be known except by testing either with a striding level, as shown in Fig. 6, or with a reversion bubble. *

Fig. 6

The design of our Precise Wye Level is such that a **supplementary bubble** can be suspended between the wyes, and, if this bubble is of the reversion type, the above requirement is not only provided for but the instrument is incidently supplied with a vial of lower sensitiveness which can be used in conjunction with a lower power eyepiece for the less accurate and less expensive work.

**See Theory and Practice of Surveying, Johnson-Smith, page 695.*
Surveying Manual, Pence and Ketchum, page 75.

The Height Adjustment for the wyes, as well as slight errors in the verticality of the spindle, are automatically compensated at the moment of observation by the "Vertical Control".

The normal position of the vertical adjustment may be secured by centering the bubble over either set of leveling screws; revolve 180° on the vertical axis, correct half the bubble displacement with the leveling screws and the other half with the vertical control. Repeat this test over both sets of leveling screws and secure in this way a verticality in the vertical axis as well as a relationship between the elements of the base bars that will keep the bubble centered during an entire revolution.

Fig. 7

If the zero of the graduated drum does not coincide with the index line above, hold the knurled head tightly with one hand and turn the graduated drum against a concealed inner spring with the other, until a coincidence is perfected. The indices of the scale above should now also coincide, as shown in Fig. 7. If not, loosen the two small set screws, which secure the scale plate, and tap the same gently in the desired direction.

The scale above the drum is intended *only* for a counter to record the number of full revolutions, and to bring the base bars back to normal position before setting up at any succeeding station. Each division on the drum represents 1/100%; one complete revolution of the drum inclines the telescope ½% and two revolutions 1%, etc. The use of stadia wires with any leveling instrument is always attended with some risk of using the wrong horizontal wire for leveling purposes. With this instrument distances may be determined with the gradienter, as explained at page 28.

The word "precise" in connection with any instrument tends to convey the impression of time wasted in locating inconsequential errors, but in this case the word is justified in the mechanical means provided to center the bubble accurately at the instant of observtion, irrespective of maladjustments in the vertical axis. This is a highly desirable requirement under the most ordinary circumstances. The use of this instrument in the field is not attended with tedium or complication, and its substantial construction as well as its reasonable price justify its adoption where difficulties with the ordinary wye level have been observed.

THE DUMPY LEVEL

Fig. 8

THIS model was introduced in England in 1845 by Wm. Gavatt, C. E., and is now in use in that country to the exclusion of nearly all other types. The objection to this type heretofore has grown out of the fact that collimation has been made subservient to other requirements; but we have found that it may be nearly as readily tested and even more accurately secured.

When the collars of a Y-level have not been perfectly made, or wear eccentric, or conical, in wye bearings that are unprotected against dust, a state of affairs exists, so far as adjustments are concerned, in which the Y-level offers little or no advantage; and the dumpy level lays claim to superiority in that there are fewer mechanical details in its construction which may have any effect upon the character of the results obtained.

In the Y-level the collimated line of sight can be brought exactly at right angles with the spindle through the height-adjustment in the wyes; but in the dumpy, no authority has yet claimed that this is necessary within a close approximation. *

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* *Some dumpies have been designed with a cross bar and two rigid upright supports. Some makers have also gone to the extreme of making the uprights adjustable in height but this impairs rigidity and adds unnecessary weight.*

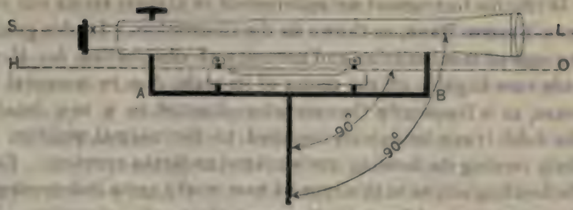

Fig. 9

In the dumpy level the vertical axis has always been considered at the basis of adjustments. Fig. 9 is intended to show an exaggerated state of affairs in which the upright, A, is shorter than the upright, B, and in which the sight-line, SL, passing through the nodal point of the objective and the intersection of the cross wires at x, is slightly inclined in the telescope tube on this account, in order that it shall be made parallel to the horizon of the bubble, HO, and perpendicular to the vertical axis.

Such a sight-line, which is coincident with neither the geometrical nor the optical axes, cannot be relied upon to remain in the horizon when focused for all distances in the field. Thus, in the sketch, if the ocular-draw is run outward for focusing on a nearby object, the cross wires will drop slightly below the original position and so impair the adjustment for parallelism. The responsibility with the maker, therefore, is to produce an ideal condition by building the telescope and vertical axis so that when the sight-line is collimated it will be at right angles to the vertical axis.

For the purposes in hand the **Level Vial** may be mounted on a base bar or upon brackets cast in one piece with the telescope tube. We prefer the latter method because weight is reduced, nothing is sacrificed to stability, and with a hinged mirror much is accomplished in the convenience of checking the bubble at the moment of observation.

The bubble is protected against accident by an outer revolving sheath, and its observation is unobstructed by any portion of the instrument. One of the main objections to using the telescope bubble of the transit for leveling purposes is due to its inaccessibility behind the standards, the vertical circle, etc.

The Reading Mirror was previously attached and adjusted to a suitable angle on a ball joint, as shown in Fig. 8, but quite recently we have changed the design in favor of a heavily constructed hinge, the tension of which can be regulated in the threaded bolt with an ordinary adjusting pin. When the mirror is closed down, it drops into a spring clip that insures it against displacement and damage.

With this instrument we recommend in highest terms our new **Three-Screw Base,** as shown in the illustration. It adds greatly to the ease of manipulation and accuracy of observation for if the bubble runs slightly off the center of its scale, due to temporal influences, as is frequently the case when directed to a new locality in the field, it can be quickly restored to the normal position by slightly turning the leveling screw nearest under the eyepiece. Each of the leveling screws in this type of base works quite independently of the other two and lends itself very nicely to this situation. To level up with the 3-screw base, turn vial parallel to any two screws and center bubble in the usual way. Now turn telescope 90° and center bubble again with third screw alone. (See page 139)

It has been contended that continued correction of this sort would affect the H. I. One division on a 20″ bubble represents 1/100 %. One who attempts to figure the one-ten-thousandth part of the distance between the leveling screw and the vertical axis is dealing either with fancy or prejudice—certainly not with practical considerations.

ADJUSTMENTS

The Horizontality of the Horizontal Wire is determined by sighting on a plumb line. If the vertical wire does not coincide, the studs of the reticule should be slightly loosened and rotated the desired amount. It is not important, however, in this or in any other leveling instrument that the vertical wire should occupy the exact center of the field.

Another method of making this test is to sight at some well defined point on the distant horizon, after all of the other adjustments are complete, and rotate the instrument on the vertical axis over an angle equal to that of the field of view. If, in passing from one side of the field to the other, the horizontal wire leaves the test point, the reticule should be rotated as before until this deviation is overcome.

To Collimate the Sight Line in dumpies of our manufacture, remove the saddle screws in the focusing pinion and withdraw the pinion from the rack. This leaves the ocular tube free to move by hand. Focus the instrument on some distant point as one would a "spy glass". Revolve the tube a half turn. If the horizontal wire moves off the test point, correct half the error and repeat the operation, using the new position of the wire for the test point until the requirements are satisfied.

Some difficulty will be experienced at first in keeping the telescope properly focused without mechanical control, but a little patience will overcome this.

It is apparent that in fixing the cross wires in the optical axis it makes no difference whether we revolve the objective (in wyes) or revolve the cross wires as directed. The chances for permanency of adjustment are better than in a wye level because the weight and "overhang" of the focusing tube are not so great.

The Bubble Tube is adjusted to right angles with the vertical axis by centering over either set of leveling screws. If using the three-screw base, swing the bubble parallel to any two screws. Revolve the telescope 180°; correct half the displacement with the leveling base and the other half with the adjusting nuts at the end of the vial tube. Previously we made this a permanent adjustment in our works, but in more recent years we have provided an adjusting screw at one end counteracted by a spring beneath the lug.

The Sight-Line is adjusted to parallelism with the bubble axis, as established above, by centering the instrument between two

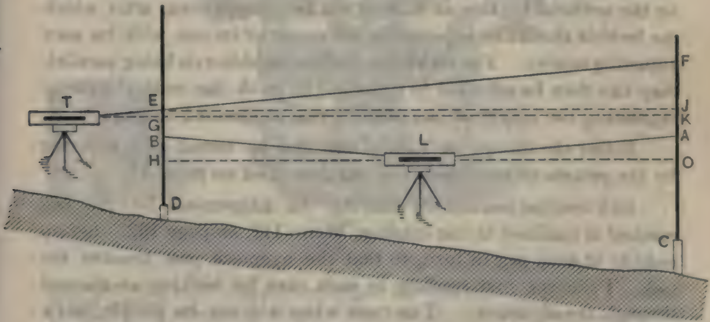

Fig. 10

pegs, C and D, say 300 feet apart. Take two readings as AC and BD. Assuming the sight-line to be inclined upward from the bubble horizon, HO, the error OA = HB if LO = LH. AC—BD therefore will be the exact difference in elevation between the pegs, without regard to the adjustments of the instrument.

Place the instrument 10 feet back of one rod at T, so that TDC forms nearly a straight line. Take the reading ED and FC. Assuming that the bubble horizon would have cut through GK, the total error would be KF and the error for the distance between the pegs is JF. From similar triangles we have:

$$KF = \frac{TK}{EJ}\left[(FC - ED) - (AC - BD)\right]$$

in which FC—ED is the apparent difference in elevation and AC—BD is the exact difference in elevation.

Subtract KF from FC to get the position of K, and, while the bubble is still centered, adjust the diaphragm upward or downward, as required, until the horizontal wire reads on K. A correction is made for the entire indicated error because the test is not made by the process of reversion.

Following out our directions for collimating the sight-line, it will not be permissible to change the position of the wires, as here indicated and as generally directed in text books. If the instrument is properly constructed, the vertical axis will be perpendicular to the collimated sight-line, and at this final stage in the process of adjustment the sight-line must cut the point K. If not, the only recourse will be to move the sight-line up or down, as suggested in Fig. 9, or report the matter to the manufacturer, for in this event the vertical axis itself needs readjustment.

In adjusting a Y-Level by this method, the collimated sight-line should also remain untouched but should be made to read on the artificial horizon at K with the leveling screws, after which the bubble should be adjusted to the center of its run with its own adjusting screws. The sight-line and the bubble-axis being parallel, they can then be adjusted to right angles with the vertical axis by regulating the height of one of the wyes, as directed on page 9.

Either the dumpy or Y-level may also be tested and adjusted by the process of reciprocal leveling described on page 37.

Still another routine for making the adjustment by the peg method is outlined in our catalog, Metro III, on page 102. Set up outside of each peg in turn, so that the eyepiece just touches the rod. Take the first reading in each case by looking at the rod through the objective. The cross wires will not be visible, but a very accurate estimate of the center of the field can be made. Sight the distant rod at each set-up in the usual manner and, if 250 ft. or more apart, subtract the tabular correction (page 36) for refraction before comparing with the first observation. Take half of the sum of the differences between the two sets of observations, so computed, and the result will be the true difference in elevation.

Being at the upper peg, set the target at this elevation plus the H.I. determined by looking through the objective in the last set of readings. The bubble being previously adjusted to the vertical axis, and now properly centered, bring the horizontal cross wire to read on the target, with the diaphragm screws, and the adjustment is considered as complete.

Note—Any of these methods may be used for the adjustment of the telescope bubble of a transit. See page 69.

ENGINEERS' PRECISE LEVEL

Made exclusively by the Bausch & Lomb Optical Co.

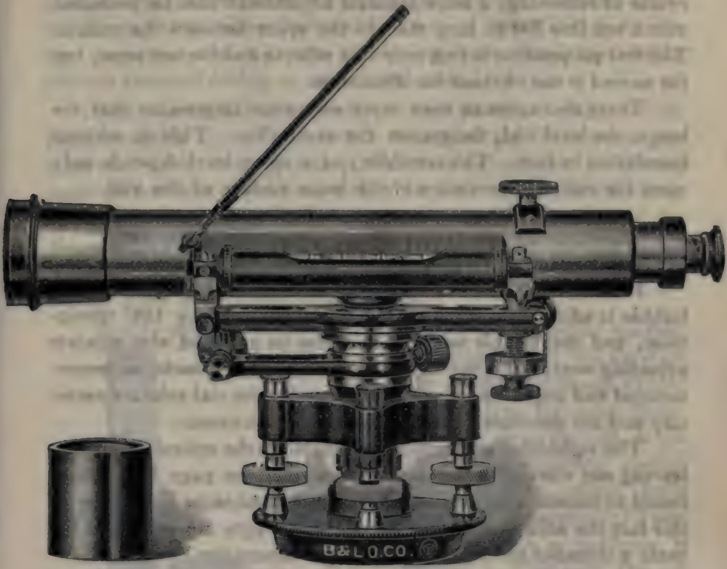

Fig. 11

THIS instrument might otherwise be known as a precise dumpy level. The design adheres generally to the dumpy model, but any leveling instrument provided with a vertical control, to supersede the leveling screws as a means of securing an accurate setting of the bubble at the moment of observation, may be known as *precise*.

In the Y-level the collimated sight-line is at the basis of adjustments. In the dumpy type, adjustments have usually been referred to the vertical axis. In the E. P. L. either plan may be pursued, preferably the first, but the adjustment for parallelism in any event must be conducted over the pegs.

The choice of instrument and method of adjustment depend upon whether one wishes to test adjustments frequently through the collars (which are in themselves the very necessity for frequent tests) or to take a little more time in the process of adjustment, in favor of greater stability of construction and the consequent greater permanency of the adjustment so made.

There exists a popular and erroneous impression that the longer the telescope, the more accurate the sight. By the same course of reasoning, a more accurate adjustment can be perfected over a test line 200 ft. long than in the space between the collars. The first proposition is true only in a relative and limited sense, but the second is too obvious for discussion.

There also exists an even more erroneous impression that the longer the level vial, the greater the sensibility. This is without foundation in fact. The sensibility of a spirit level depends only upon the radius of curvature of the inner surface of the vial.

ADJUSTMENTS

If the vertical axis is chosen as a basis of adjustments, the bubble is adjusted to it with the "vertical control" by 180° revolutions, and the bubble need not therefore be provided with separate adjusting nuts; but if the sight-line is adopted as a basis, as recommended and urged by us, adjusting nuts for the vial tube are necessary and are therefore furnished with each instrument.

The sight-line, as carefully collimated to the optical axis before leaving our works, by the method described on page 18, will be found to maintain its position with great persistance. In view of this fact the adjusting screws for the diaphragm are concealed beneath a threaded ferrule, as shown in the illustration.

Conduct the adjustment by either of the methods described for the dumpy level, and in taking the final step bring the horizontal wire to read on the target, as set, by use of the vertical control. If the bubble runs off center, correct the entire error with the adjusting nuts at the end of the vial tube.

By reference to the inserted diagram the sight-line, SL, and the bubble horizon, HO, are now parallel. Figure 12 will further

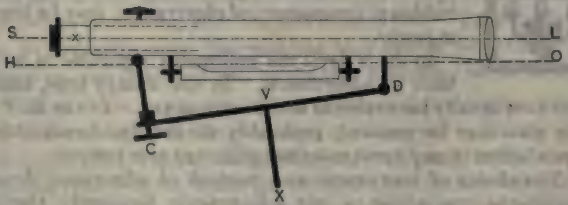

Fig. 12

show how the sight-line can be brought into the true horizon by use of the vertical control, C, without regard to the adjustment of the vertical axis, VX.

To Adjust the Vertical Axis to Verticality, however, level up over either set of leveling screws, revolve the instrument 180° or thereabouts, correct half of the bubble displacement with the leveling screws and the other half with the vertical control, C. Repeat the operation over both sets of leveling screws until the bubble remains centered during an entire revolution.

This process secures a normal position for the vertical control and the vertical axis with respect to the axis of the bubble, and for accurate work it is necessary that this adjustment should be accomplished and maintained within the limit of a few divisions on the bubble tube. The H. I. is determined by the position of the pivots at D, and it will be readily seen that if the axis VX is not truly vertical, or very nearly so, the elevation of the point D will change when the instrument is reversed.

Having determined the normal position of the vertical control, as directed above, the **zero of the graduated drum** should coincide with the index blade. If not, clamp the set screw just above the vertical control and turn the drum itself against a concealed inner spring until a coincidence is effected. Clamping the set screw may slightly disturb the bubble, but it will center itself again on being released. The little index line on the indicator blade, as located by us, should now cut across the top of the graduated drum.

Fig. 13

In using the instrument, the V. C. should be constantly used to center the bubble at the instant of observation without moving in one's tracks. By this means, residual errors in the adjustment of the vertical axis are quickly overcome, and, as shown in the diagram, the sight-line and bubble axis can be brought

into a truly horizontal position without reference to the leveling base. Taking into account the desired accuracy in the work, some discretion should be exercised and some limit should be set as to the amount of bubble error which may be rectified by this means without a readjustment of the leveling base.

A Bulls-Eye Level is placed on the base bar, CD, at the point V. This is to be used only for the rough preliminary setting of the vertical axis before the final precautions are taken in setting up the instrument for work. It is assumed that if the vertical axis is adjusted with the bulls-eye level, it will be set with sufficient accuracy so that observations can be taken immediately by use of the vertical control.

This instrument can be set up and operated without the necessity of the operator changing his position. This is a very great convenience for work in swamp lands, or under other unstable conditions, and the telescope is short enough to give the operator a wide sweep of field without shifting his weight.

To use this instrument as an ordinary dumpy level, very carefully secure a normal position for the vertical control, as previously directed, then clamp the thumb screw shown in Fig. 13 just at the right and just above the graduated drum. This will keep the bubble axis and the sight-line permanently normal to the vertical axis.

In this case, as recommended for the dumpy level, more rapid and satisfactory results may be obtained by use of the three-screw base, as suggested in the illustration.

PERCENTAGE GRADIENTS

The graduated drum may also be used for the observation or establishment of percentage grades in railway or highway work, or for the determination of distances by the gradienter method.

Example

To establish a 1.65% grade, for instance, secure a normal position for the V. C., as directed above, and get the H.I. with respect to the starting point by any convenient means. Each division on the drum represents 1/100%; a single revolution, or 50 divisions, ½% and two revolutions, 1%, etc. To lay off 1.65%, therefore, turn the drum in elevation or depression as desired, three revolutions plus 15 extra divisions.

The telescope will now occupy a position in an inclined plane, making a slope of 1.65% with the horizon, and will cut any position in the field, in the general direction of the telescope, in this proportion with respect to the peg over which the instrument is located. In other words, the line of sight occupies a position parallel to the grade which is to be established. The H.I. is a constant for that particular set-up. Let it be assumed that this is 4.857 and that a down-grade is being run.

The rod may be held at any point on the grade line without regard to its distance from the instrument.

Suppose the first reading to be	7.154
H. I.	4.857
Fill,	2.297

Again :

The H. I. being - - -	4.857
Suppose the second reading to be	2.715
Cut,	2.142

Signal Code

For convenience in the transmission of signals on gradient work we insert opposite a system devised by R. S. Beard of the C. B. & Q. R. R., as published in the *Eng. News,* April 21, 1913.

KEY

A. Rod Up or Give Line—Wave a small horizontal circle high over head with hand or rod.

B. Move to Right or Left—Move hand at shoulder height in direction required; slightly for a small move, arm length for a big move.

C. Plumb Rod to Right or Left—Extend arm vertically and swing slowly in direction required.

D. Set a Hub or Give a Turning Point—Hold any visible object horizontally overhead with both hands.

E. Set on This—With point of rod on tack, swing top of rod in small circle.

F. All Right or Go Ahead—A half-circle swing of one or both arms overhead.

G. Come to Point Signaled From—A full-circle swing with either arm.

H. Go Back—Face to the right of man signaled to, wave both arms up and down, raising one while dropping the other.

I. Can't Get You—Describe an "L" or inverted "T" by a vertical motion, followed by a horizontal motion of the arm.

J. Move Target Up or Down—Move hand up or down in direction required. Motion of hand means move 0.01; half-arm, 0.1; full arm, 1.0 ft.

NOTE—When calling numbers use any prearranged word for eleven.

Fig. 14

For Use as a Telemeter, the gradienter of the E. P. L. does not differ materially in theory from the stadia except that the constant (c + f) is not considered, and the datum point, is at the pivots, D, (see Fig. 12) instead of at the anterior focal point, or center of the instrument. In such an instrument as the one here considered, which must of necessity work close to the horizon, no correction need be made for inclination of sight.*

Example

Level the instrument and suppose the horizon reading to be,	6.738
Turn graduated drum two revolutions to the right or left and suppose the second reading to be - - - -	3.126
Interval, 1:100 - - -	3.612

or 361.2 ft. from the pivots of the instrument.

On the speculation that any number of subdivisions of the drum, (n), are turned and that an arbitary number of feet, (f), are covered on the rod at the distance (d), we have :

$$d : 10,000 :: f : n$$

Having a value of 1/100 of 1%, each subdivision of the drum will encompass 1 ft. on a rod at 10,000 ft. distance. Suppose 117 divisions cover 5 ft. on a rod at a certain distance. Substituting in the above equation, we find that d = 426.5 ft. from the pivots.

Tables on pp. XXIV and XXV will be found a convenient reference in this connection. Percentage equivalents in degrees and minutes can be obtained by comparing the first and third columns, thus: $1.6\% = 0°55'$, or $1\% = 34'23''$.

In this connection let it be remembered that 34′23″ is equal to the stadia interval of 1:100. If one has no stadia wires in his instrument, let him measure the intercept subtended by this angle on a rod held horizontally and multiply the factor by 100. No constant is to be added in this case.

**Any Stadia Reduction Table will show that corrections for horizontal distances due to inclination of sight do not become appreciable until the interval between 0°26′ to 0°42′ is reached. The first is equal to 0.756% and the second, 1.222%.*

COMPENSATION LEVEL

THE fundamental principle of adjustment which underlies most instrumental tests is the one of reversion, by which the doubled error can be measured and rectified. It is the simplest and most effective method known and has always dominated the ideals of speculative thinkers on this subject.

Nearly all the extensive discussion on leveling methods has grown out of attempts to rectify instrumental deficiences by reducing rather than removing them. In the transit small errors of spacing may be quite successfully overcome by repetition, but the only method of eliminating instrumental errors in leveling instruments heretofore has been by the equalization of the forward and back-sight.

This recourse is always tedious and frequently impossible. Every precedent and every impulse have suggested the necessity for a telescope through which telescopic observations might be taken in both directions—forward and backward, through the same tube. Such a device is now an accomplished fact, and the principle of reciprocal vision has made it possible to check the sight-line directly against the bubble axis by the simple precaution of turning the telescope end-for-end and applying the eyepiece to the other extremity.

The adjustment and operation of this self-contained instrument are very simple.* Two achromatic objectives, of equal aperture and focal length, are carefully centered and immovably mounted at opposite ends of the telescope tube. Their optical axes are collimated to one and the same straight line in a special apparatus and the terminations of the optical axis are marked by cross lines, engraved with a diamond point, on the outside surface of each objective. Each objective, in turn, performs the function of a glass diaphragm for the other objective.

The engraved cross lines supersede the usual spider web and are indestructible. If dust particles collect on the surface of the objective, they will also appear in the field of view and should be dusted off with the camels hair brush provided for this purpose. The cross lines are microscopic and do not interfere in the slightest with telescopic vision.

The eyepiece is removable and attachable to either end of the telescope and the focusing is accomplished, for sighting in either direction through the instrument, with an interior negative lens which maintains all of the required optical properties. The lens system is symmetrical and the process of focusing provides for homologous positions for the interior lens.

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* *See Article by Dunbar D. Scott, Vol. LXXVI, Trans. A. S. C. E. 1913, p. 1172, etc. A copy of this paper will be mailed on request.*

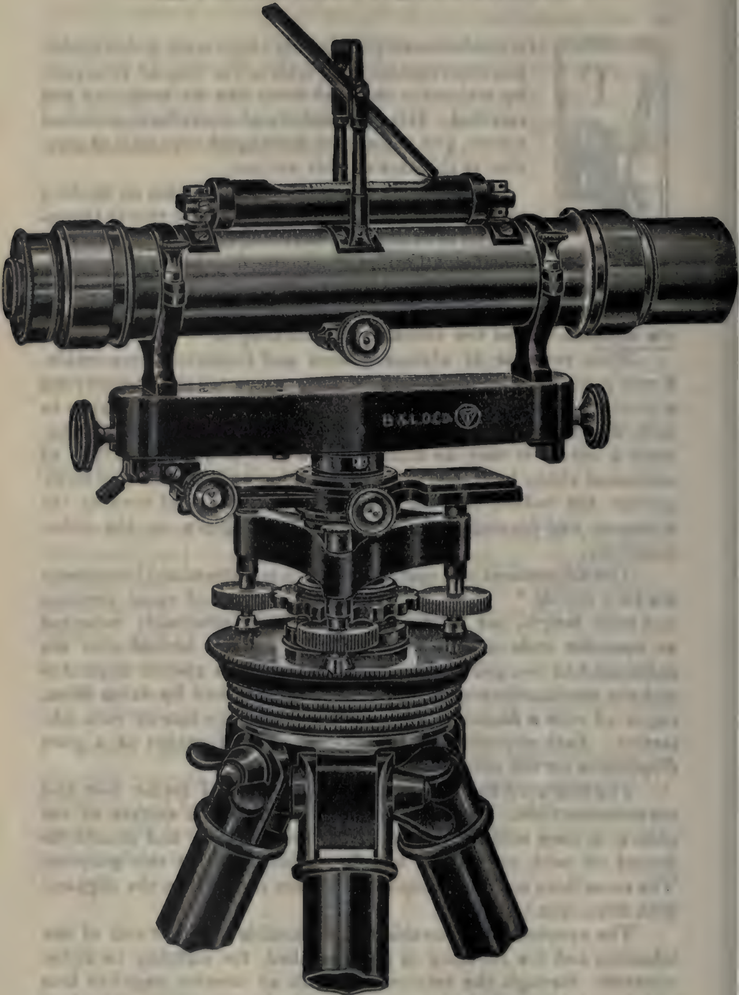

Fig. 15

Bausch & Lomb Compensation Level; Reciproscopic Telescope, Cryptic Focus, Self-contained Adjustments.

The means employed to concentrate an image produced by a fixed lens upon an immovable diaphragm may be understood by an inspection of Fig. 16, which is reproduced from a discussion on the tele-photo lens by Baker.* The principle is not new but Mr. H. Wild of Jena, was the first to use it for this purpose in 1909.**

Naturally the image of a distant object would be produced at the principal, or shortest, focus of the objective, but interposing the negative lens at the proper position throws the image back the

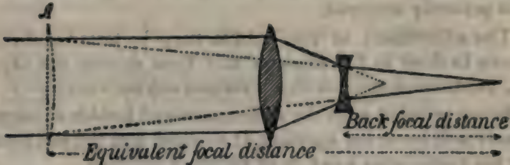

Fig 16

required distance, producing the same effect as though an objective of equivalent focal length had been racked out to the position shown by the lens in dotted outline at A.

In effect, we have two telescopes combined in one, and avowedly two optical axes which must be collimated to coincidence so that sights can be taken in either direction along the same straight line. This adjustment is the only one upon which the accuracy of the instrument depends, so that it is permanently fixed by us. That the operator may reassure himself, however, in regard to the collimation adjustment, a set of improvised wyes are provided; but they are not intended for use in the bubble adjustment.

There being no collimation adjustment to concern the operator, proceed with the **Bubble Test** as follows:—

Set up instrument firmly with one set of leveling screws in the general direction of the rod.

Bring bubble carefully to the center of its run with the Vertical Control by turning the knurled head at either end of the base bars, and take a reading on a rod about 100, or 150, ft. from the instrument.

Tip the mirror in its pinions, remove ocular, leave vertical axis clamped but open key in the clamp-and-tangent movement; revolve telescope end-for-end and re-engage the key in the seating provided. Replace ocular and re-focus on rod.

If the bubble has run off center bring it back with the Vertical Control and take a second reading.

If the readings are identical, the bubble is in adjustment.

**Thick Lens Optics*, A. L. Baker, 1912, p. 90.*

***Zeitschrift fur Instrumentenkunde*, Nov. 1909, p. 329.*

If there is a difference, the True Horizon for this H. I. at this station will be determined by taking the mean of both observations.

In this case, bring the telescope to read on the True Horizon, thus determined, when the line of sight must be truly horizontal. Bring the bubble to the center of its scale by its own adjusting nuts and the test is complete.

As a final test, reverse the instrument once more and, centering the instrument on the horizon line a second time, see that the bubble is perfectly centered.

The adjustment of the bubble may be tested and rectified in this way in three to five minutes any time during the day, on any sight, but these directions do not assume any such necessity.

Being satisfied with the collimation and bubble adjustment, the engineer will leave the eyepiece at one end and the sun shade at the other for continuous work as in ordinary practice.

The instrument illustrated in Fig. 15 has a telescope 25 cm in length and an aperture of 30 mm. It has been designed to combine unusual compactness with utmost accuracy, and the construction is such that the telecope can be instantly transformed from the erecting to the inverting type by a simple exchange of eyepieces. We are prepared to supply two orthoscopic (inverting) oculars producing 18 and 26 magnifications, also an erecting eyepiece (not shown in the illustration) with a special sun shade to balance, giving 20 magnifications.

For convenience in manipulation, **the normal position for the Vertical Control** should be determined, as directed on pp. 15 or 23. This will be most conveniently accomplished by swinging the instrument exactly 180° by use of the **release key** in the clamp-and-tangent movement. In the lugs of this mechanism are small set screws by which 180° revolutions can be very exactly accomplished. This arrangement is designed to bring the instrument directly back on the rod when reversed for double observations.

The normal adjustment of the Vertical Control is one of convenience and not of importance. The collimation and bubble axes being parallel, they are brought, by this means, into a plane that is horizontal in all directions. The work is further facilitated by restoring the bubble to a normal position with the vertical axis before setting up at any station. Taking this precaution, it should not be necessary to turn the fine motion screw over half a turn, in either direction, to center the bubble accurately at each observation and to overcome, in this way, slight errors in the adjustment of the vertical axis.

If the focusing device is unimpaired and the bubble in working order, perfect results can be obtained with this instrument in remote frontier districts or elsewhere, on the assumption that the instrument

has sustained some injury that has destroyed the collimation adjustment. In the event of such extreme necessity, proceed as follows:

Bring the bubble to the center of the scale (mirror removed) and suppose the first reading to be . 5.274

Loosen the collar clips and revolve the telescope on its own axis a half turn, when the second reading may be . 5.258

Correcting for collimation error, while looking in one direction through the telescope; the mean result will be 5.266

Restore telescope to original position. Remove ocular; revolve on vertical axis by use of release key; adjust ocular to opposite end, level up a second time and suppose the third observation to be . 5.242

Turn telescope again upside-down and, presupposing the collimation to be in error also in this system, let it be assumed that the fourth reading is 5.274

Correcting for collimation error, while looking in the opposite direction through the telescope, the mean result of the second set of observations will be 5.258

The discrepancy shown in the two averages indicates that the bubble is out of adjustment but a mean of the two averages, or 5.262

will be a perfect result deduced by corrections possible in the instrument itself by the infallible process of reversion — an accomplishment impossible with any other type of level heretofore constructed.

The instrument's first claim upon the attention of the engineer is the facility with which the adjustment for parallelism may be tested and the accuracy and speed with which it may be rectified. An open, unbiased examination into those parts which might be suspected of infidelity develops the conclusion that, one by one, they may be reduced to negligible quantities.

The influence of the new cryptic focus on minimizing the errors of lens displacement on collimation adjustment is especially remarkable. It is is discussed on p. 78.

The Coast Survey Level

With the beginning of this Century, the U. S. C. & G. S. discarded the wye-construction in favor of a precision dumpy type, of which our Engineers' Precise Level is a simplified modification. To overcome the effects of temperature the telescope and the outer tubular encasement were cast in nickel-steel and nickel-iron and wherever the vast experience of the geodecists of this department dictated radical and scientific departures from conventionalities, they were adopted with conspicuous success. It is said America leads the world in the accuracy, rapidity and economy of precise leveling.

Fig. 17

Outside of the powerful telescope and the method of observing the bubble, by prismatic reflection, at the instant of observation, the **chambered level vial** is the dominating feature of its construction.

A view of the chambered vial is given in Fig. 17. There is a glass partition between the larger and smaller chambers with a tunnel connection in the under side. The fluid is sulphuric ether, which finds its equilibrium quickly but is more susceptible to temperature changes than alcohol. When, in very hot weather, the air bulb shrinks within the normal lines, there is made an exchange of fluid and air through the orifice by a little clever manipulation.

These vials are sensitive to one or two seconds of arc and uniformly ground.

We have manufactured this instrument to official specification since its inception, but the question of precision leveling is rather too involved for exposition here. For the student who desires to pursue the topic we append a list of references :—

U. S. C. & G. S. Report, 1900, Apx. 6, p. 521

Trans. A. S. C. E., Vol. XLV, 1901, pp.127-175

U. S. C. & G. S. Report, 1902, Apx. 4; *ibid* 1903, p. 200

J. F. Hayford in Eng. News July 2, 1903

E. M. Douglas " " " May 17, 1906

C. H. Lee " " " Sept. 17, 1908

H. W. Maynard " Cornell "Civil Engineer" May '09 and Nov.'10

H. C. Mitchell " Eng. News, Mar. 23, 1911, p. 356

C. M. Cade " Eng. & Cont. Nov. 29, 1911, p. 591

Prof. W. H. Burger, Trans. Ills. Soc. Eng. & Surv. Vol. 26, p. 141

"Geodetic Surveying" Prof. E. L. Ingram 1911, p. 153-162

W. E. Jessup in "Wisconsin Engineer" Mar. 1913

W. N. Showalter in National Geographic Magazine, June 1914

Bausch & Lomb Metro III Catalog, p. 116 etc.

Curvature and Refraction

The phenomenon which produces the mirage in very hot or extremely cold climates also affects more or less, under all climatic conditions, barometric pressure and rectilinear vision in solar observations, in hypsometry and precise differential leveling.

When a luminous ray passes obliquely from one air stratum to another, the incident ray is bent or refracted from its original direction. In a general way it may be said, if it passes into a denser medium, it is bent nearer to the perpendicular or normal line NM, and conversely, if into a rarer medium it is bent away from the normal line.

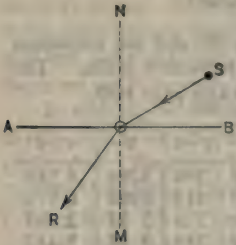

Fig. 18

In Fig. 18, let S be the source of light and AB the line of separation between two strata of different density. The angle of incidence is SON and the angle of refraction is ROM. Assuming that the region ARMB is the denser, the beam SO will be refracted in the direction OR; but had the region ARMB been superheated and therefore less dense, the angle ROM would have been greater than the angle SON.

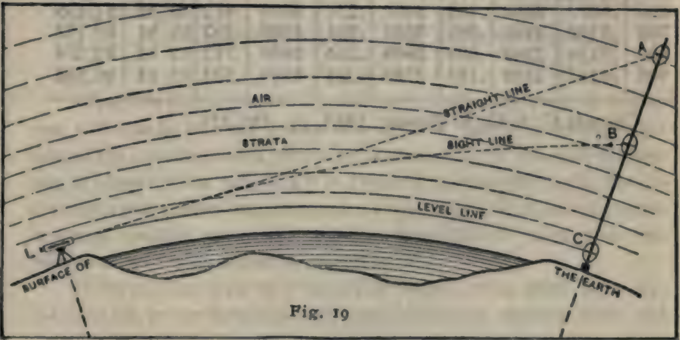

Fig. 19

The effect of normal atmospheric refraction is to make distant objects appear higher than they really are because a ray of light, starting from the target as shown by the arrow at B, Fig. 19, is bent downward and enters the telescope at L as though it had come in a straight line from A.

Curvature tends to increase rod readings, as between C and A. This error can be determined with reasonable accuracy. It varies directly as the square of the distance and may be computed by the formula :—

$$.667 \times D \text{ - miles}^2$$

Refraction tends to decrease the above correction as shown in the figure. The curve of refraction depends somewhat upon the density of the atmosphere, but the empirical valuation fixes the radius at seven times that of the earth. The combined resultant effect of curvature and refraction (BC in the figure) is the one usually considered and is represented by the formula :—

$$C + R = .57135 \times D - \text{miles}^2.$$

In the following table, the distance D, and the correction C + R are given in feet, except where M represents miles. The correction is always a minus quantity. (See also page 165).

D	C+R	D	C+R	D	C+R	D	C+R	D	C+R
100	.0002	1000	.021	2400	.118	3800	.296	5200	0.554
150	.0004	1100	.025	2500	.128	3900	.312	5280	0.571
200	.0008	1200	.030	2600	.139	4000	.328	2 M	2.285
250	.0012	1300	.035	2700	.149	4100	.345	3 M	5.142
300	.0018	1400	.040	2800	.161	4200	.362	4 M	9.141
350	.0025	1500	.046	2900	.172	4300	.379	5 M	14.282
400	.0032	1600	.052	3000	.184	4400	.397	6 M	20.567
450	.0041	1700	.059	3100	.197	4500	.415	7 M	27.994
500	.0051	1800	.066	3200	.210	4600	.434	8 M	36.563
550	.0062	1900	.074	3300	.223	4700	.453	9 M	46.279
600	.0073	2000	.082	3400	.237	4800	.472	10 M	57.135
700	.0100	2100	.090	3500	.251	4900	.492	11 M	69.133
800	.0130	2200	.099	3600	.266	5000	.512	12 M	82.274
900	.0166	2300	.108	3700	.281	5100	.533	13 M	96.558

Consult C. & G. S. Report, 1883, pp. 289-321.

Reciprocal Leveling

Correction for curvature and refraction, as well as for instrumental inaccuracies, can be mechanically overcome by reciprocal leveling, which consists in finding the difference in elevation between two points by two sets of observations.

Set up between the points and near to one of them. Take readings on each rod with the bubble carefully centered. Set up in the same relative position with respect to the second point and take two more observations. The true difference in elevation is the mean result between the differences in two sets of readings.

In running level lines across rivers or estuaries, where the effect of refraction is variable and considerable, this method has a special application. Establish bench marks on each shore as shown in Fig. 20.* With the instrument at B, take a B.S. on the rod A, then a F.S. on rod C. The lower dotted line, representing a true horizon, the computed elevation of B. M., C, will be too low by the distance between the dotted line and the point *b*.

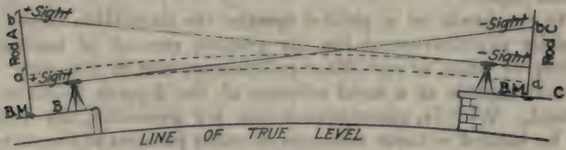

Fig. 20

With the instrument at C, the computed difference in elevation will also be in error by the amount $a' \, b'$ which is the algebraic sum of the curvature and refraction correction. A mean of the two sets of observations will be a true result if the two intervals, due to curvature and refraction, are equal. To be certain that this influence is not a variable quantity, the best practitioners have used two instruments for simultaneous observations.

If the regular type of Y-Level is used for this class of work, the variable power eyepiece with magnifications ranging from x18 up to x36 will be found desirable to accommodate the optical properties of the instrument to atmospheric and light conditions. If our Precision Y-Level is employed for this work, the inverting type of telescope will always give the best attainable visual result and a choice of oculars giving a range of magnification between x33 and x46 is the best adaptability of available means to the required purpose.

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* *See article by F. W. Koop on Precise Leveling in New York with Coast Survey Level as made by the Bausch & Lomb Optical Co., Trans Mun. Engrs. N. Y., 1913, pp. 75-151.*

THE PLANE TABLE

FROM a mathematical standpoint the plane table cannot be regarded as an accurate instrument, for there are both theoretical and practical errors in the method of using it which offend the nicely trained instincts of the field engineer; but the accuracy of any map is limited, not so much by that of the field instruments as by the protractor and the proportions of the scale. The fact that a point can be oriented by an alidade as accurately as it can be located on a small scale map has given rise to the growing conviction that for certain classes of topography in open country the plane table and stadia method is not equalled by any other system.

The scheme of plotting the map in the field and of sketching in all details, as the work progresses, without elaborate mental or written notes and with fewer computations or located points, probably more than offsets, in time and expense, the extra field work and the weather hazard.

In the hands of a skillful operator the plane table is not excelled by any instrument for the graphic control of horizontal position by polar or rectilinear coördinates, and is the only instrument capable of a rapid solution of the 3-point problem in the field. With it, cumulative errors are overcome, for points may be located without reference to other portions of the survey.

Field work may be conducted by the methods of Radiation, Intersection, Resection, Traversing, Radio-Progression, or by the Two- or Three-Point Problems. †

Traverse methods consist essentially of leveling and orienting the table, drawing lines in the direction sighted and scaling off distances by which each station becomes the origin of a separate polar coördinate system. Intersection methods, however, are generally more accurate and can be more reliably checked, precisely as triangulation in geodetic control is more accurate than instrumental traverse.

Rules for Graphic Control. *

1. Occupy the known point first.
2. Measure base lines twice in opposite directions. The base line plotted on the plane table to field scale should not be less than two inches in length.
3. No location is good unless made by three intersections.
4. Intersection angles at important stations should not be less than 30°. Two lines intersecting produce a minimum error when they meet at 90°; three lines at 60° or 120° etc.

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* *D. M. Higgins in Economic Geology, Dec. 1913, pp. 729-751.*
†*See also D. L. Reaburn in Eng. News, Mar. 26, 1914*

5. Orientation should be accomplished by at least two back sights and should be checked occasionally to guard against accidental displacement of the table.
6. Three-point locations should be made by the three nearest available points checked, if possible, by a fourth, and the final orientation should be made on the farthest visible station whether or not it was used in the location.
7. Errors in orientation should be corrected in the field, but errors of position with reference to long check sights are better adjusted in the office by slightly shifting the points to accord with check sights.

Details of Procedure in the Quadrilateral Method.

Referring to Fig. 21, first set the point a_1 carefully over the corresponding end of the base line. A plumb need not be used unless the

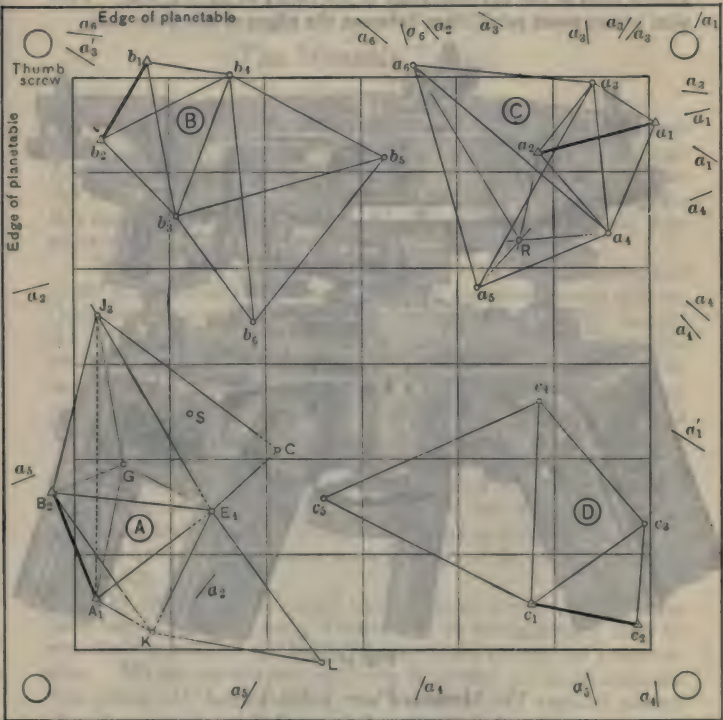

Fig. 21.—Various Methods of Graphic Base Extension after Higgins. A, The Auxiliary Point Method; B and C, Quadrilateral Method; D, Triangle Method. Scale, 2 in. = 1 mile.

work is on a very large scale. Even in large scale maps the plumbing arm is rarely used except on short or important sights. The table should be so oriented that the survey line shall occur on the paper in the general desired direction. Sight carefully at a_2, with the edge of the ruler nearest the operator coincident with a_1, and draw $a_1 a_2$. Also draw a short production of this line on the margin as indicated. With this kind of margin control, the maximum permissible error of setting the base rule should not exceed three minutes of arc or 4.6 ft. per mile. Without disturbing the position of the planchette, sight a_3 and a_4 and draw the respective courses accurately to scale.

If the extension lines, like a_1 and a_3, occur near the corner, so that they may not serve as accurate subsequent guides, draw auxiliary guides lines at the opposite edge of the ruler, as at a'_1 a'_3. This plan presupposes parallelism between the edges of the base rule.

Fig. 22

The Mensula Plane Table Tripod
with Ball Bearing Azimuth Adjustment
as made exclusively by the
Bausch & Lomb Optical Co.

Move the outfit to a_2 and with the fiducial edge of the rule resting against a fine needle in the point a_2, and coincident also with the point a_1, revolve the planchette until the sight line hits the station a_1. Great care in plumbing would be a waste of time. An error of half the width of a 15 x 15 traverse board, in the sight of a mile, plotted to a scale of 2 inches on the map, would scarcely be discernible. The instinctive tendency, however, is fortunately toward greater accuracy of orientation and with care, when the last station is occupied, the converging lines should meet in the same needle hole.

If it were necessary to locate some point as at R, the three-point problem should be applied, as from the three nearest visible stations, and the orientation conducted upon the most distant station in the survey that is visible.

The Mensula Tripod

The most elaborate type of tripod and head mechanism is shown in Fig. 22. It is usually associated with the thought of the Coast Survey, although it is used by State and Municipal Departments for the more careful field mapping. We have choosen to designate this model as the Mensula Tripod in honor of J. Prātorius, who so named the plane table when he invented it in 1590.

In our model, the circle of rotation is 12 in. in diameter and contains a ball-bearing race of 125,5-mm phosphor bronze spheres. Otherwise the construction is all brass and weighs 28 lbs. without the planchette which is not shown. This weight would at first seem excessive, but for work in windy country some weight will be required to offset wind pressure on the planchette. Some topographers hang a canvass sack by three hooks from the tripod legs and fill the same with stones to accomplish this purpose.

Experience has shown that constant wear on the leveling screws induces a certain amount of lost motion which would cause annoyance in accurate orientation. We have met this situation by supporting the threaded shanks in a split and tapered bushing which can be adjusted by two check nuts, above and below, as indicated in Fig. 22.

The details in the larger topographical surveys, which have been based upon a strong framework of triangulation, may be accurately controlled and located by this more substantial construction. Where topographic mapping is carried on simultaneously with plane-table triangulation, the availability of vertical control should be considered in the selection of the initial station. Vertical angle elevations have relative values dependant upon the size of the angle measured and the distance between points.

The Johnson Tripod

Fig. 23

It was found that the heavy 3-screw leveling mechanism, with slow motion screw of the Mensula Tripod, carried the refinements of construction beyond the requirements for some topographic mapping. The Johnson Spherical Socket, introduced in 1887, provides for rapidity and sufficient accuracy in both leveling and orientation. It is shown in perspective in Fig. 23 and in section in Fig. 24.

There is no tangent screw for slow motion in azimuth in this construction, for it is understood that the leverage furnished by the outer edges of the planchette accomplishes all practical requirements.

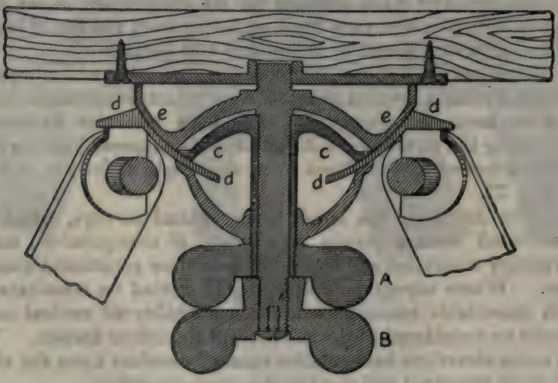

Fig. 24 (From Breed and Hosmer).

The graphic horizontal projection of survey lines is not appreciably affected by slight inclinations of the planchette, particularly if the alidade is provided with a control bubble to secure a coincidence of the index of the vertical arc with the nadir.

In Fig. 24, the upper clamp, A, controls the axis in apposition while leveling the planchette, and the lower clamp, B, controls rotation. In setting up the table loosen both clamps. When the circular level on the alidade is centered, clamp A, and when the table is oriented, clamp B. Check the sights after clamping to overcome possible torsional stress in the tripod legs. It is advisable, in careful work, to loosen the clamping bolts of the tripod legs slightly then re-clamp to secure stable conditions before completing the process of orientation.

Traverse Tripods

Considerable is saved in both weight and expense by simplifying the head mechanism, as in our **No. 84 Traverse Tripod,** and mounting the same on the regular split legs commonly supplied

Fig. 25

with a 4-in. Transit. This makes it possible also to supply extension legs with which the planchette can be leveled and for the further purpose of condensing bulk in shipment. This tripod is sufficiently subtantial to support the largest planchette and standard alidade, and with a special adapter, or with a separate metallic head, it can be used for a 4-in. Transit or a 4½-in. Tachymeter.

The head mechanism is rigidly designed with an entirely new method of connecting the planchette with the tripod. By unscrewing the larger knurled head the tension is sufficiently released so that the key-bolt may be entered and nicely seated in the plate of the planchette, as indicated in Fig. 25. The board can then be rotated by hand until oriented, and the large knurled head tightened to make the connection more secure.

The **No. 85 Traverse Tripod**, a modification of Mr. Gannett's or Mr. Bumsted's design, as illustrated in Fig. 26, has spindle legs

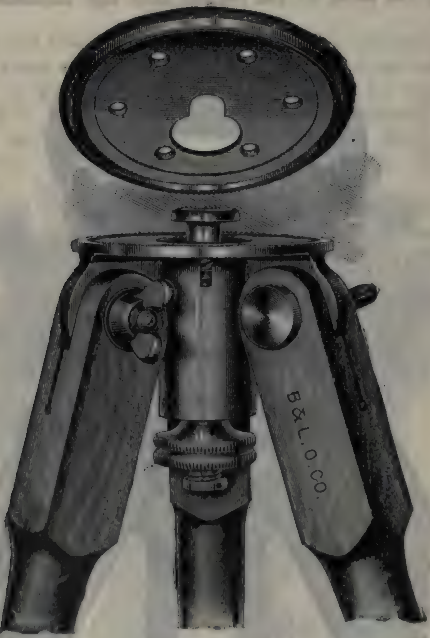

Fig. 26

and weighs only 5 lbs. It is especially adapted for use with a 15 x 15 planchette and the so-called Graton Traverse Alidade with sight vanes, declinatoire and small circular bubble, for preliminary surveys in remote districts. The 15 x 15 planchette is large enough for a township plotted 2 in. per mile.

The planchette is secured to the head mechanism in much the same manner as just described. If the knurled head is turned to the left, the key-bolt is released; if turned to the right, the connection is tightened. The planchette may be revolved on the circular seating and is held in position there by friction. There is no means of leveling the table except by carefully choosing the position for the spindle legs.

Fig. 27

The Planchette

The planchette is a drafting board of suitable size made of well seasoned flawless pine and constructed in sections to prevent warping. It is rarely larger than 24 x 30 inches (6.1 x 7.6 dm). The intermediate size is 18 x 24 (4.6 x 6.1 dm) and the traverse size, 15 x 15 (3.8 x 3.8 dm). In the under side is inserted some sort of mechanical provision to fasten the same securely to the tripod, as indicated in the upper portions of Figs. 25 and 26.

For attaching the paper to the board various methods are in use. When a survey extends at great distance in one direction, the paper has been fed continuously from rollers beneath the board. That portion which is in use can be held in position by tension from the rollers or by spring clips. Thumb tacks are not recommended* because they interfere with the unrestricted movement of the alidade.

What is generally conceded to be the best practise is to countersink, in the outer edges of the planchette, a system of six or eight threaded sockets and fasten the paper with screw-tacks that are set down flush with the surface. † These keep the paper from lateral displacement and prevent it from being ripped from the table in high winds. Our planchettes are usually made in this way, but we are prepared to supply the nickle plated spring clips on request. The declinatoire, which may be mortised into the edge of the planchette, is shown in Fig. 27 and described on page 48.

* *Plane Surveying, A. E. Phillips, C. E., Ph. D., 1910, p. 175.*
† *Topographic Surveying, H. M. Wilson, 1900, p. 177.*

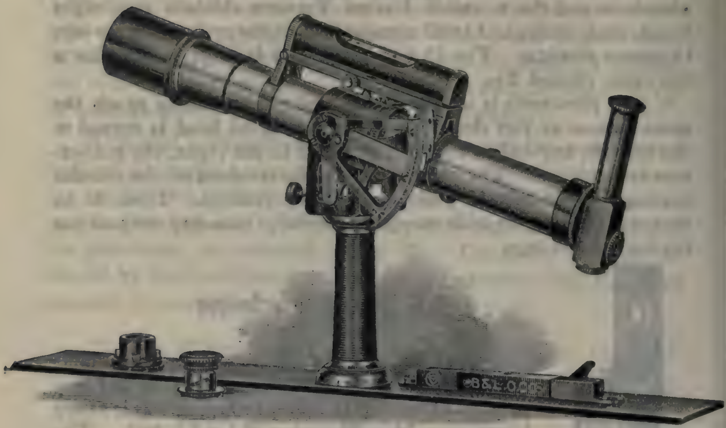

Fig. 28

No. 90 Standard Topographic Alidade with Special Diagonal Eyepiece for Direct and Indirect Sighting.

The growing demand for alidades of quality and adaptability, for a wide variety of purposes, is responsible for the various modified types which we now manufacture. The standard model shown above is the composite result of the most expert experience in the field and shop. The full circle and the attached bubble tube of the older types have given way to the edge reading arc and the removable striding level. The telescope is mounted centrally in low standards over the base plate for, though there be a slight error of coincidence between the sight line and the fiducial edge, it is a constant that will not affect the accuracy of the more careful work.

It was previously thought that the telescope should be mounted in an eccentric bearing so that it could be reversed in altitude in a plane coincident with the edge of the ruler; but plunging the telescope is unnecessary either in work or adjustment, and a sight line that is removed 1½″ (38 mm) from the ruling edge will not introduce perceptible errors except in rare sights of a few yards distance.

The Telescope is equal to that of the best theodolites. It is of the inverting type, x23 or x32 as desired, provided with our new interior focusing system, revolves longitudinally in the axis sheath against 180° stops, and in elevation in a substantial standard whose lower corners are rounded for convenience in handling.

The Duplex Diagonal Eyepiece shown in the engraving is a new design in which one may take observations either directly through the principal ocular, when it is thrown into alignment with the collimation axis, or indirectly through the cathetus extension,

by simply sliding the tube downward to the stop-catch, as indicated in the illustration. The planchette should be set about elbow height, but continuous bending over it is a tiresome occupation. Frequently the eyepiece prism is used so that the operator may look directly downward when taking sights. This new device makes such an expedient the more desirable, particularly as one need not remove his hat. When this eyepiece is used, a little heavier sunshade is employed. The extra ocular shown in Fig. 28 can be substituted whenever desired.

The Vertical Arc is of 60° duration with a single vernier centered at the 30° mark. This makes it impossible to confuse small angles of elevation or depression; 30° must either be subtracted from the observed result or *vice versa*. Occasionally there is supplied, beside the vertical arc, the Beaman stadia arc as shown above and in Fig. 65, p. 115.

The Pillar is aluminum, cord wound, either 3½ or 4¼ inches high, as preferred, and the **Base Rule** is a brass blade 16 × 2¾ or 18 × 3 in., as desired. The contact surface of the base rule has been previously covered with a light brown drafting paper which could be cleaned with a pencil eraser, or fine emery paper, but we have recently adopted a white enamel finish, which can be washed and is very satisfactory.

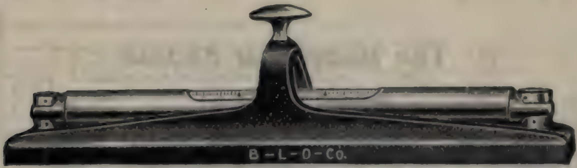

Fig. 29

Level Vials are still preferred by some topographers mounted in coördinate horizontal planes on the base rule, or countersunk in the planchette, but their adjustment against even slight inequalities of the base rule has been found to consume much unnecessary time. We are prepared, however, to furnish this system in place of the circular level if so specified.

The Bulls Eye Level has no gauged sensibility but is amply accurate for the intended purpose and does not distract the operator with indicated errors of no concern. At any rate, as a measure of economics, the leveling device should be mounted on the base rule for it may be thus applied to any planchette. If one wishes to secure unusual accuracy in leveling the planchette for any reason,

a **Bar Level**, shown in Fig. 29, is unsurpassed in convenience and adaptability. It may be mounted in the case with the instrument and serve other useful purposes. The Bar Level may be adjusted by placing it on the planchette; bring bubble to center by tapping the board, reverse bar level end-for-end and correct half errors in the board and the adjusting screws of vial mount until the bubble remains centered in both positions.

The Declinatoire, or box compass, is indispensable, for the reference line is usually the local magnetic meridian. It is generally mounted on the base rule in preference to inserting it in the edge of the planchette, but we are prepared to furnish it for this purpose, as shown in the marginal cut Fig. 27, if so desired. It is provided with one of our tubular compass needles with sapphire mounting, as enthusiastically approved by Government officials and many others who have used them. The Declinatoire has no adjustment except the clamping device and the balance of the needle, which must be adjusted by the user for different latitudes. To do this remove the metal cover that envelopes the central portion of the box and, sliding the glass window back, the needle becomes accessible. The mounting and clamping device employed insures the needle against dismounting when unclamped. (See also page 140).

The Micrometer Alidade

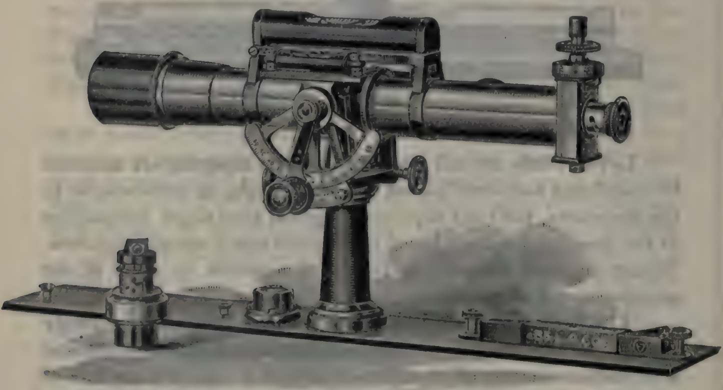

Fig. 30

No. 92 Telemetric Alidade with Interchangeable Oculars.

In the stadia method of estimating distances we have a fixed interval in the telescope corresponding to 1/100 of the focal length of the objective and a variable interval on the rod depending upon its distance from the instrument. The angular value of the field of the average telescope, depending upon the focal length, aperture and magnification, varies between about two and four degrees, but the stadia interval, if accurately spaced, will occupy exactly 34′ 23″ of said field. The apex of this angle is at the anterior focal point, and the intercept on the rod constitutes the subtended base. The sum of the angles at the base is therefore equal to 179° 25′ 37″, and considering the privileges we are at liberty to take with small angles, one of those at the rod may be designated as a right angle. In this case we have $B = P \cot a$. The *nat cot* 34′ 23″ is 100, but any other spacing that could be included in the field might be used and distances, from the anterior focal point, calculated in the same way.

Then let us mount a single wire on a special movable staging in the focal plane, that can traverse the whole field, and measure the movement of the wire on the graduations of a drum head, the exact angular value of which, for any particular telescope, may be determined by experiment. Fig. 31 is a modified reproduction from the *Eng. News*, Vol 65, No. 16, where R. H. Sargent, U. S. G. S. gives expert testimony regarding the efficiency and adaptability of this instrument for both detailed and reconnaissance surveys. The

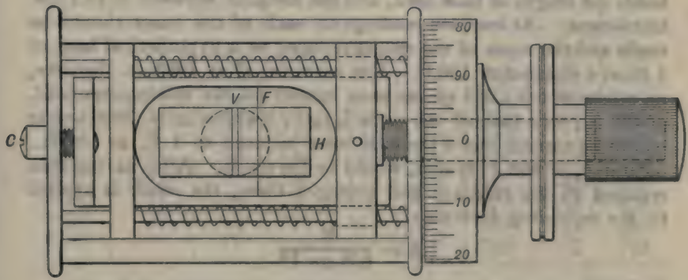

Fig. 31

filar micrometer diaphragm consists essentially of a fixed staging, with a horizontal wire at H and a twin vertical wire at V, so disposed that the collimation adjustment can be accomplished independently by rotating the telescope in the axis sheath and making the necessary correction with the adjusting screw, C.

When the movable wire, F, occupies the middle space between the double fixed vertical wires, the zero of the graduated drum should coincide with the index. If not, the drum is turned on the shank of the screw until these conditions are fulfilled. The movable wire, F, can be made to traverse the entire field, as indicated by the

dotted line, and by revolving the telescope in the axis sheath measurements of this nature can be taken either in the vertical or horizontal plane. If observations are to be taken between the targets on an ordinary leveling rod, held vertically, the micrometer box is turned with the graduated drum at the top; but if observations are to be made at great distances, between specially prepared signals, the whole telescope is to be turned until the graduated drum is at the right hand and the focusing pinion underneath.

To find the drum values in seconds of arc we give directions from Mr. Sargent's article as follows:—

Place carefully measured test bases at 500, 1000, 1500 and 2000 ft. from the center of the alidade and not from the anterior focal point of the objective, as in the stadia method. At the 500-ft. station two stakes should be set 5 ft. apart and another between them at 3 ft. from one end, thus giving three bases—one of 5 ft., a second of 3 ft. and a third of 2 ft. At the 1000-ft. station, the extreme stakes should be set 8 ft. apart with an intermediate stake 5 ft. from one end. At the 1500-ft. station the extreme pegs should be 10 ft. apart with sub-bases at 4 ft. and 6 ft., and at the 2000-ft. station the longest base should be 15 ft. with intervening bases of 7 ft. and 8 ft.

Adjust the hair by means of the micrometer head until it covers one end of a base, then revolve the head until the other end of the base is covered, noting the number of divisions passed over on the head, the length of base used, and the distance between it and the instrument. At least ten readings on each of the bases should be made and the mean of the results reduced to a hypothetical base of 1 ft. at a distance of 100 ft. For example, if at 1000 ft. distance, on a base of 6 ft., 365 spaces were turned on the micrometer head, at 100 ft., on a base of the same length, theroretically, 3650 spaces would be turned; but if the base were reduced to 1 ft. at 100 ft., $3650 \div 6$, or 608.33 spaces, would be the result. After having thus reduced all the readings and averaged them, the result is applied in the following formula:

$$d = \frac{B}{RH \sin 1''}$$

in which

d = value in seconds of arc of one division of the micrometer head,
B = length of reduced base,
R = number of divisions turned on micrometer head, and
H = distance from instrument to base.

Therefore $$d = \frac{1}{100 \times 608.33 \sin 1''}$$

$$\begin{aligned}
1\ \log &= .00000 \\
100\ \text{colog.} &= 8.00000 \\
608.33\ \text{colog.} &= 7.21586 \\
\text{Colog.} \sin 1'' &= \underline{5.31443} \\
\text{Log. } d &= 0.53029 = 3.''391
\end{aligned}$$

C is the constant, or ratio, to be found for each instrument.

$$C = \frac{1}{d \sin 1''}$$

Substitute the value of d from the first equation, and the formula becomes:

$$C = \frac{H\,R}{B}$$

$$\begin{aligned} B &= 1 \text{ ft., } \textit{colog.} &&= .00000 \\ H &= 100 \text{ ft., } \textit{log.} &&= 2.00000 \\ R &= 608.33 \text{ ft., } \textit{log.} &&= \underline{2.78414} \\ C &= \text{Constant, } \textit{log.} &&= 4.78414, = 60833 \end{aligned}$$

By transposition in the last equation we have:

$$H = \frac{B\,C}{R}$$

Example: Base, or B, = 6 ft.
Divisions on the head, or R, = 365.
Find the distance from the base, or H.

$$H = \frac{B\ C}{R} = \frac{6 \times 60833}{365} = 999.999 \text{ ft., or } 1000 \text{ ft.}$$

When the instrument is used for running lines of comparatively short sights, a table similar to the following, computed for each instrument, may be used.

Miles	BASES							
	1	2	3	4	5	6	8	10
0.21	56	112	168	224	280	336	448	560
0.22	53	107	160	214	268	322	428	535
0.23	51	102	153	204	256	307	409	512
0.24	49	98	147	198	245	294	392	490
0.25	47	94	141	188	235	282	376	471
0.26	45	90	136	181	226	276	362	453
0.27	43	87	131	174	218	261	348	436
0.28	41	84	126	168	210	252	336	420
0.29	40	81	121	162	203	243	324	406
0.30	39	78	118	157	196	235	312	392

The figures 1, 2, 3, etc., shown at the head of the table, are the length, in feet, of bases which are used, either upon a stadia rod or established upon the ground. In the left hand column, the quantities are 100ths. of a mile. The figures in the body of the table represent the number of divisions read on the micrometer head. In explanation of the use of the table, let us suppose that a base of 5 ft. has been used and 254 divisions have been recorded from the micrometer head. Following down the column headed 5, the nearest figure to 254 is 256. By following this line to the left we find that 0.23 miles is the distance. If a closer determination is required, it may be found by interpolation. For longer distances and

greater bases than might be shown in such a table, the computation is quickly made by applying in the formula last given in the computation above.

The bearing of a horizontal base constructed on the ground must be ascertained in order that the angle between it and the line of sight from the instrument may be known and used in the computation of a corrected base. If the work is being executed by means of a plane-table, with the table in orientation, the direction of the base may be drawn on the sheet. If a transit is being used for the work, the bearings of the base and the line of sight may be determined, either directly by means of the compass, or by the angle between them and a line of known direction.

When occupying a point at which it is desired to use this base for ascertaining the distance between it and the point occupied, the correct length of the observed base may be obtained in the following manner:

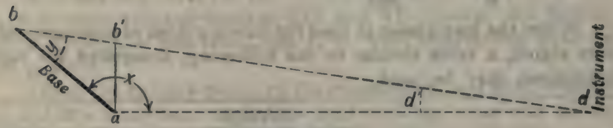

Fig. 32

Suppose *a b* represents the base established and *a d* and *b d* the lines of sight, from the station occupied, to the base. Since the angle *d′*, at the instrument, is so very small, if a perpendicular were erected from *a* to the line connecting the instrument at *d*, with the center of the base, the angle which it would make with *b d* at *b′*, so closely approaches 90° that it may be considered as such. Orient the plane-table as closely as possible, either with the magnetic needle or by inspection; then, by placing the alidade upon the point *a* on the paper, sighting to the signal *a* on the ground and drawing the line *d a* upon the paper, we have the angle, *x*, between the base and the line of sight. Plot upon the paper the line *a b* as long as possible. For instance, if it is 10 ft. on the ground, if possible, plot it 10 in. on the paper. Then from *a* erect a perpendicular to *d b*. Then *a b′*, measured by the same scale as that by which *a b* was plotted, will be the length of the corrected base, or B, to be used in the fore-going formula. If a table of sines is available, and also a protractor, the angle *y* may be measured and the length of *a b′* found by multiplying the sine of *y* by the length of *a b*. The same principle applies to a signal erected vertically, which is above or below the station occupied.

In case a horizontal base above or below the instrument is used, and it is desired to reduce the reading to the horizontal distance, the following formula, which can be easily derived, should be used:

$$H = \frac{(ab)\ C \sin y \cos v}{R}$$

in which:

H = Horizontal distance from instrument to base,
ab = Base as erected,
C = Constant,
y = Angle between base and line of sight of instrument,
v = Vertical angle at instrument.

Conclusions.

The stadia ceases to be useful when the wires cut over the top of the rod. Using only the half interval of 1:200, a 13-ft. rod would be useless beyond half a mile. The superior advantage of the filar micrometer lies in the fact that very much greater distances can be estimated directly from the center of the instrument with any convenient length of base. For traversing streams or trails, where much of the field of view is cut off with brush or some kind of impedimentia, any small portion of the rod that is visible may be used as a basis of calculation. Extremely long sights may be taken without relying upon the rod-man since cairn signals can be constructed, at known distances apart, and left standing for subsequent back sights. * Such bases, 500 ft. in length, have been used in estimating distances of 25 miles or more. The accuracy is equal to that of the stadia, ranging between $\pm$ 0.1 and 0.5 of 1%.

The Vertical Arc on this instrument is a segment of 120° with graduations on the face, or side, and a double vernier reading to minutes. The attached magnifier has a metallic pointer which can be used, while reading, as an indicator to superimpose the mark indicating the whole number of degrees.

The Striding Level, as in the standard model, is removable; 4¾ in. long and has a sensibility of 60″ per each 2 mm division engraved on the glass.

The Control Bubble, with protection sheath and tangent adjustment, is considered indispensable, not only on this instrument but on our No. 95 Frontier Model as well, which is a miniature reproduction of the standard model No. 90. Slight inclinations of the planchette in the direction of the sight cannot affect the true value of the vertical angle if the index of the vernier is brought into the nadir, or horizon, as the case may be, before the observation is taken. If the striding level is broken or lost, the planchette may be conveniently leveled by the control bubble after being properly adjusted to the sight line. It is 2 in. long and has a sensitiveness of 60″.

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* *See J. D. Craig, D. L. S., on the "Canada-Alaska Boundary Survey" in Eng. Rec., May 16, 1914.*

The Fiducial Edge should be straight. Draw a line against the straight edge and turn the alidade end-for-end. If the straight edge coincides perfectly with the test line, the requirements are satisfied.

The Base Rule Bubbles are to be made parallel to the plane of the table or the under surface of the base rule. Set the alidade on the planchette, mark a line along the fiducial edge and bring the bubbles to the center of their scales. Turn the alidade end-for-end along the guide line. If the bubbles remain centered, the adjustment is complete. If not, do not undertake to readjust the planchette but simply correct half of the error in the adjusting screws of the level vials.

The Bulls Eye Level has now generally superceded the graduated vial because it is conceded to be accurate enough for the intended purpose. Whereas level vials are ground to cylindrical curves, the inner surface of the circular bubble is that of a sphere of long radius. Usually the normal position of the bubble only is shown by a small circle, but if graduations were supplied they would occur in series of concentric circles. These bubbles are adjusted by three small screws, not shown in Fig. 33, operating against a circular spring plate in the base. Once adjusted in the factory these bubbles rarely have to be rectified, but in case of this necessity place the alidade upon a plane surface, which is known to be level in all directions, and tighten the screw toward which the bubble seems to creep.

Fig. 33

The Telescope Axis must be adjusted to horizontality in order that the vertical wire shall travel in a vertical plane. This adjustment cannot be conducted with such care as in the transit, for it is too difficult to level up the base with such accuracy or stability, and no adjustment for the horizontal axis can be conducted with any certainty unless the base rule is perfectly level across its shortest dimension. The plumb line test is therefore sufficient. Carefully level the planchette and place the alidade along a well defined mark. Place a plumb line in the field of view and revolve the telescope to check against it. If the vertical wire deviates to the right, for instance, reverse alidade along the guide line and test on another plumb line. If in this case the sight line deviates an

equal amount to the left, the test will show that, while the plane table is not horizontal in the direction of the telescope axis, the axis itself is correct.

The standards are cast in bronze and the axis in No. 1 red metal so that fretting is reduced to a minimum. The adjustment as made in the factory, therefore, may be considered as reasonably permanent. With the vertical arc on one extension of the horizontal axis and the vertical clamp on the other, it would be difficult to provide an adjusting block, especially when the most experienced operators have found such a contrivance unnecessary.

Collimation for both of the cross wires is tested and rectified as in the Y-level, as directed on p. 6. The telescope is mounted revolvably between 180° stops in the axis-sleeve for this purpose. At the under side of the sleeve is a plunger exerting just enough pressure to prevent the telescope from turning on its axis in the process of focusing. All alidades are made, by common consent, with the inverting telescope only. If the cross wire has to be moved in making this test, it should therefore be moved in the direction which is apparently necessary.

It was thought heretofore that if the vertical cross wire were brought into the apparent center of the field, the practical requirements in the case had been fulfilled. No telescope, however, is capable of accurate vision, in both planes, at all distances in the field, until both wires are properly collimated. With all the conveniences at hand, there is no excuse why the vertical wire should not also be accurately located in the optical axis by the process of rotation as directed above.

The Striding Level, which is removable and reversable, is supported on two red metal collars that do *not* constitute the axis of longitudinal revolution, as in a Y-level. When instruments leave our works, however, they are trued up to the same axis of rotation. There is very little chance for wear in the carefully protected rotation sleeve and the collars are subject to little or no wear, so that we may reasonably assume that this adjustment will contemporize itself with the life of the instrument.

The rapid test therefore is to level the telescope by the striding level, then turn the striding level end-for-end on the collars. If an error is indicated, correct one half with the tangent screw and the other half in the striding level. To accomplish this, turn the set screw, in the crotch of one of the wyes, with a screw driver. As in the wye level this secures parallelism between the bubble axis and the contact points on the collars, but does

not guarantee parallelism in this case with the line of sight unless the maker has provided for it. The certain test is the peg method. As a matter of fact this is a dumpy level proposition that is being conducted on a wye-and-collar basis to no very serious purpose. If, in the peg method, the collimated sight-line is brought into the predetermined horizon on the rod and the bubble will not remain centered on being reversed, mount the striding level with the release button on the side of the telescope nearest the vertical arc, center the bubble carefully to those conditions and thereafter use the striding level in this position only. This method disregards the collars entirely as a means of adjustment, and a disparity in their diameters is of no concern.

The Wind Adjustment, or lateral adjustment, tends to bring the axis of the vial vertically over the telescope axis. Temporarily remove the clamping post in the hub of the telescope and revolve the vial from side to side on the collars. Correct the error as described for the wye level, p. 8. This adjustment is not extremely important for, while the clamp allows an unrestrained seating on the collars, there is still too little opportunity for side movements to cause serious displacements in the bubble.

The Vernier of the Vertical Arc must be adjusted to the limb when the telescope is in a horizontal position, as indicated by the striding level. If the indices do not coincide, slightly loosen the screws that secure the vernier plate and gently tap the scale until a coincidence is perfected; then tighten the set screws, keeping the reading edges in contact. The zero of the vernier is set at the 30° mark so that small angles of elevation or depression will not be confused.

The Control Bubble should be perfectly centered when the zeros of the vernier and the vertical arc are coincident. If not, center the bubble with its own adjusting screws. When instruments are provided with this attachment the index coincidence, provided for in the paragraph above, is accomplished by turning the tangent screw that regulates the movement of the vernier scale and control bubble mount. When the coincidence is perfected, bring the control bubble to the center of the scale with its own adjusting nuts. In this position the axis of the bubble is not only normal to the index of the vernier but is parallel to the line of sight.

In an extremity, the telescope may be considered level, therefore, when the zeros of the arc and vernier scale correspond and the control bubble is in the center of its run. In this way the control bubble may be delegated to perform the office of a striding level that has been damaged or lost.

The Pioneer Alidade

This alidade, as designed for W. H. Boyd, Can. Geol. Survey, is a replica of our standard model reduced in size and weight about one-third. The base plate is 12 in. long, the arc has a radius of 45 mm and the inverting telescope is usually furnished with 25 mm aperture and 16 magnifications.

Like all our alidades the diapragm is furnished with the stadia wires without additional charge. Whatever may be said of the achievements and possibilities of the stadia principle, it is always understood to be accurate up to the requirements of average plane table work.

For those who prefer the gradienter method, however, we have also supplied, with this and with the next described instrument, a gradienter drum with celluloid index as specified by Eugene Stebbinger, U. S. G. S. (See top of p. 123).

This alidade, in connection with our No. 84 Traverse Tripod, is highly recommended for mountian reconnoissance and for economic railroad location. On prairie land, ten miles a day has been encompassed on contour work. In *Eng. Rec.*, July 20, 1912, Prof. E. L. Griggs, Univ. of Ga., reports on the use of the plane table for highway construction and extensive relocation.

The Miniature Alidade

Manufactured exclusively by the B. & L. O. Co.

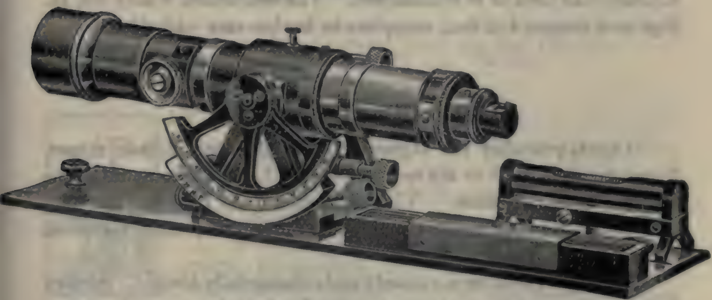

Fig. 34

In *Economic Geology,* Vol. VII, No. 7, 1912, C. H. Wegemann says, "A very compact telescopic instrument is that known as the Gale Alidade, which was planned by H. S. Gale, U. S. G. S. in 1909 and is unexcelled where a light compact instrument is required."

The arc is one of 120° duration, numbered from 0° to 60° each side of a center index, but on account of the absence of a 3½-in. pillar, such as is used with the Pioneer type, not all of the arc can be used. The ocular and objective mounts strike the plate at —18°

and $+28°$, so that if sights of greater inclination are to be taken these extremes are to be laid off by noting some point in the landscape; then, after setting the vernier back to zero, build up one end of the instrument with any substantial bolster like a note book and read the increment beyond the first limit.

Such angles are used in connection with tables to calulate difference in elevation by the hypsometric method. A constant check should be kept on elavations, so determined, which in geological mapping should agree within 5 ft. ordinarily, or 10 ft. on great distances. On large scale maps covering areas above one mile in any direction, a correction for curvature of the earth is to be applied to reduced horizontal distance by use of the table on p. 36.

The standard of this instrument is so low (2-in.) that there is not enough room for direct vision. The ocular is therefore supplied with a permanently fixed total reflecting prism, and all observations are taken by looking straight down into the exit pupil. This same idea is carried out in a large and better way with our duplex diagonal eyepiece, as shown in Fig. 28 and made only by us.

The adjustment and use of this instrument do not differ materially from the larger models, except that every sight is to be taken while looking downward into the fixed prismatic eyepiece. The base rule is $2\frac{3}{4} \times 11$ inches and may be divided into tenths and fiftieths of an inch or in millimeters. The instrument is only $3\frac{1}{2}$ in. high and weighs $4\frac{1}{2}$ lbs., complete in leather case with sling.

The Traverse Alidade

This is probably the highest grade sight-vane alidade extant. It was made in 1907 to the requirements of L. C. Graton, now of the Harvard Geol. Museum. The aluminum base rule is 7 in. long and usually divided into decimal inches though frequently furnished in the metric scale.

The sight vanes are accurately and substantially hinged. As they fold down one of them clamps the needle of the $3\frac{1}{2}$-in. declinatoire. In April, 1913, we mounted, in an aluminum case, a small circular level on the base rule at Mr. Graton's request. The instrument weighs 5 oz. (140 grms), folds up very compactly and is packed in a leather pouch which is to be slung at the hip of one's girdle.

THE TRANSIT

Fig. 35—Showing general equipment with Vertical Clamp, Telescope Bubble and Vertical Arc, covered under the No. 043 4-in., No. 053 5-in. and No. 063 6-in. Transit.

THE TRANSIT is an instrument made use of by operative engineers to project alignments, to observe or lay out angles in the horizontal or vertical plane, to estimate distances, establish grades, run levels and erect perpendiculars.

It is made up essentially of three component parts; the leveling base, which contains the outer socket of the compound center, the graduated horizontal plate which is mounted on an intermediate socket known as the azimuth axis, and the alidade, carrying the vernier scales, the standards, the telescope and vertical limb, which is mounted on an inner spindle known as the vertical axis. The compound centers are, therefore, more or less intimately related to every working portion of the instrument. The graduated circles have been aptly designated as the "brain" and a similitude has been drawn between the telescope and the "soul" of the instrument, but the compound centers are the constitution in which all of the organic laws of adjustment and operation have their origin.

The Portable Transit, for land surveyors and engineers, is an American invention dating back to 1831, when a low power telescope was mounted, on plain truss standards over the dial of a surveyor's compass to replace the sight vanes. This precedent has dominated all subsequent practice and manufacture for, while the vagaries of the compass are now well known, that feature is still a conspicuous part of the modern instrument.

Our Transits are made in three sizes: the four, five and six-in. models conforming closely to conventionalities except where, in certain mechanical or optical details, we have been able to improve the construction by adding to strength, accuracy or reliability.

The Telescopes are provided with the objective draw tube focus now in common use. The collimation is secured for all distances without special adjusting screws or interior springs. The focusing pinion, by common consent, is at the top; convenient for either right or left hand, erect or inverted.

The Standards are of symmetrical and unique design, being reinforced at the base with a wide contact surface that permits two dowel screws from beneath and one through the center from the top. This is the most substantial system known to us. The lateral stiffness is remarkable. (see also p. 130).

The Vernier Reflectors are a special translucent milk glass which not only reflects but transmits light for the ample illumination of the vernier plates. The vernier windows are flush with the plate so that dust or rain can be readily removed.

The Plate Bubbles are in the compass box protected against accident and sudden temperature changes. They are mounted on a swiveled base bar that offers unusually sensitive control and can be adjusted by a capstan head screw from the outside. Removing the plate bubbles from the zone outside the compass ring leaves more room for vernier openings and contact surface for the standards.

The Leveling Screws are provided with dust caps and dust guards. The lower guard can be unscrewed and removed entirely for mining work so as to insure a maximum tilt in the leveling head. (See Fig. 123 p. 197.)

The popularity of the 6-inch Transit has been sustained mainly by those who have believed that the length of the needle was a measure of its efficiency. We have received inquiries for 8-inch Transits with this conviction the evident purpose.

The telescopic attainments possible with the 6-inch size, however, are plausible causes for preferment. In the matter of increased stability, due to weight and the improved telescope qualifications, the 6-inch model stands pre-eminent among the portable instruments. Stability does not necessarily increase with weight because it is regulated somewhat by stiffness in the tripod head and legs; but with the variable power eyepiece the magnification can be increased to $\times 27$ without noticeably impairing the rectilinear, planar or luminous qualities of the field.

ADJUSTMENTS of the transit instrument are comparatively simple when the geometrical elements and their relationship to one another are fixed in mind. The vertical axis is a vertical line passing through the spindle, intercepted at the plates by a horizontal plane and terminating in the line of collimation at its intersection with the horizontal axis of the telescope. This point in the telescope, at the intersection of the horizontal axis, the collimation axis, and the vertical axis prolonged, is the virtual center of the instrument.

The Plate Bubbles are intended to establish verticality in the compound centers and incidentally to level the plane of the graduated plate. If these two functions are to be performed simultaneously, it is evidently incumbent upon the manufacturer to build a vertical axis that will remain permanently at right angles to the vernier plates, and to construct journal bearings in the standards, moreover, that can be secured at right angles to the vertical axis prolonged.

The Azimuth Axis must also be built at right angles to the plates or, in revolving the instrument, the vernier scales will appear to rise and fall above and below the reading edge and cause parallax of vision in consequence.

The Sight Line and the axis of the telescope bubble must not only be parallel but the construction must be such that the sight line will revolve in a vertical plane, passing through the vertical axis when focused for all distances in the field.

When the Graduations are accurately spaced and perfectly centered with respect to the azimuth axis and the other conditions expressed above are fulfilled, then the instrument is, in the essentials, qualified for reliable work; but the responsibility of the manufacturer is so great in this accomplishment that he must share with the surveyor to some extent the further accountability as to the accuracy of the field work.

The "*Engineering Record*" for Sept. 2, 1911 acknowledges editorially that the amount of time and money wasted annually in every city, in reaching compromises over boundaries of improperly surveyed tracts, is very large. One of the steady incomes of the legal profession is derived through the adjustment of failures on the part of the surveyor. Errors of this kind arise from faulty computations, careless procedure, or instrumental deficiencies which the surveyor either does not recognize or correct.

We have heard of young surveyors who were afraid to investigate their instruments because they did not understand them, and others who, having once set up an instrument, will not touch it again at that station whatever the indicated errors. There are

others also who, having received an instrument in apparently good condition, allow the screws to rust in. *

Prof. Walter Harris of Princeton, Prof. J. M. Porter of Lafayette, and Prof. H. K. Vedder of Mich. Agri. Coll., among others who are alert to the practical necessities of the case, have given us the assurance that no student is allowed to "pass" until he has stood a satisfactory examination in adjustments. Indifference to the necessity or reason for instrumental adjustments has caused many embarassing situations in the field. When a polygon will not close, either the graduation is defective or the proper precautions in the routine of work have been slighted.

In starting the adjustments set up the tripod firmly on a substantial footing. The wing nuts at the top of the tripod legs should be screwed home tightly so that when held in the air the leg will not drop of its own weight. Much depends upon the fit of the bolts, both in the wooden forks and in the brass lug of the tripod head. The tripod itself should not be too light, merely to save weight.

The survey, no doubt, is affected by unstable conditions in the tripod. With all of our portable azimuth instruments we offer an optional choice between the No. 75 Tripod with 60-in. legs, weighing 9½ lbs., and the No. 76, weighing 11½ lbs. The No. 74 Tripod, designed for the Reconnoissance Transit, the Mountain Tachymeter and the No. 110 Compensation Level, has 57-in. legs and weighs only 7¼ lbs. To secure greater stability, many engineers prefer to order, for small instruments, the No. 75 Tripod with a **Reducing Ring**, shown in Fig 108 which expedient, in fact, makes it possible to use one tripod for two different sized instruments.

I. To Adjust the Tangent Axes of the Plate Bubbles Perpendicular to the Vertical Axis and Parallel with the Graduated Plates.

(a) The bubble vials are usually supplied in pairs and placed at right angles on the plate. Unclamp the vertical axis and turn the alidade so that each bubble will lie parallel with one set of leveling screws. If the 3-screw base is used, turn one bubble parallel to any two screws; the other will then lie in a co-ordinate plane controlled by the third screw. It is advisable to adjust only one vial at a time, but not much dexterity is required to adjust them simultaneously. Bring the bubble to the center of its scale with the leveling screws. Revolve the instrument 180°, preferably by vernier, on the vertical axis and note the error of displacement. Correct half with the leveling screws and the other half with the adjusting stud of the vial mount.

The leveling screw threads are usually too coarse to get perfect results at first, but center the bubble carefully and swing back to

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* *Manual of Adjustments, H. C. Ives, 1896, p. 2.*

the original position. Correct the error by halves as before and continue the test for each bubble until they remain centered during an entire revolution. It is customary to adjust one bubble over one set of leveling screws and the other over the second set. Both sets of leveling screws must be brought into commission or the vertical axis will not be adjusted to verticality in both planes.

(b) **A second method** of adjusting the plate bubbles, and indeed a more accurate one, may be accomplished with the more sensitive bubble suspended beneath the telescope. Swing the telescope vial over either set of leveling screws; set the vernier plates to zero and clamp both the vertical and azimuth axes. Level the telescope bubble with the leveling screws, after having set the vertical arc to the index of the vernier.

Revolve 180° on the vertical axis; correct half the error indicated by the telescope bubble with the leveling screws and the other half with the tangent screw which controls the telescope. Turn the instrument back 90° and, using the other set of leveling screws, bring the bubble to the center of its scale. If the first test and correction were perfectly made, the bubble will have been made normal to the vertical axis and a further adjustment of the bubble should not be required; but revolve 180° again and correct half errors as before if necessary.

By repeated tests over both sets of leveling screws the telescope bubble will ultimately remain centered during an entire revolution. The final careful test will be made over either set of leveling screws, and when verifying over the other set half corrections should not be necessary unless eccentricity of centers is evident, in which case the bubble cannot be reconciled to both sets of leveling screws.

The vertical axis, in the manner prescribed, has been adjusted to perfect verticality by temporary use of the telescope bubble, whose relationship with the sight line has no influence or bearing whatever in this case. If the plate bubbles are now adjusted to the center of their scales, the adjustment is complete.

Clamp the vertical axis and open the azimuth axis. Revolve 180°, as nearly as may be judged by the eye, and note the position of the plate bubbles. If there is a noticeable deviation, eccentricity of centers is evident. In all our portable instruments we aim to keep this error of centering in the compound spindles down to a few seconds of arc and in instruments intended for finer reading and accurate triangulation, this error must be reduced to zero. Plate bubbles cannot be kept in adjustment if the compound centers do not revolve about the same axial line.

II. To Test for Eccentricity of Centers.

Open both vertical and azimuth clamps; tie the clamping mechanism at the edge of the plates with a cord to some fixed object, and with the telescope bubble still centered turn the lower plate with the tips of the fingers in a direction against the pull of the cord.

Watch the telescope bubble and estimate the deviation in seconds of arc on the scale. If the bubble moves a third of a division on a 30-sec. scale during a complete revolution of the underplate, a total eccentricity of 10 seconds in the centers may be assumed. This will mean a plus or minus error of 5 seconds, which is just as likely to be compensating as cumulative in running levels.

The plate bubbles, as a matter of precedent and not as a matter of reason, usually have a sensitiveness of about 60 seconds per division. Some plate bubbles are less sensitive than this so that an eccentricity of 10 seconds in the centers will scarcely be noticeable, except when tested with the telescope bubble as explained above.

An error of this size is not worth taking into account as to its effect on reading horizontal angles, particularly when the probability of much more formidable errors in linear measurement is considered; but for leveling purposes it will be apparent that the accuracy of the operation is not so much dependent upon the length of the spindle as upon the care with which it is centered.

We insert a table of errors calculated on a basis of 1000—let this be feet, or meters, or any unit of measure. The errors are given in percentages and will be proportional for other lengths of line.

Error	Per 1000	Ratio
2″	.0097	1 : 103100
4″	.0194	1 : 51550
6″	.0291	1 : 34370
10″	.0485	1 : 20650
20″	.0970	1 : 10310
30″	.1457	1 : 6870
60″	.2909	1 : 3440

By reference to the table it will appear that an error of 30 seconds in the centers will produce a probable plus or minus error of .073 per 1000 which scarcely comes within the limits prescribed for ordinary good leveling. Let it be borne in mind, however, that the effect of such an error upon reading minute graduations need not be so seriously contemplated.

III. Parallax.

The function of the objective is to produce a minute aerial image of the field at the plane of the cross wires, and the coincidence of the cross wires with this microscopic image depends upon the accuracy with which the objective is focused.

The office of the eyepiece is merely to magnify the image of the field so as to produce a visual image some twenty to thirty times its original size. These conditions make it first necessary to

focus the eyepiece carefully upon the cross wires, then to reproduce an image of the field on this plane with the objective.

Turn the telescope away from the sun and toward the sky, or simply hold a piece of white paper inclined in front of the objective so as to throw a diffused sky light into the telescope. Focus the eyepiece with the worm slot until the cross wires appear distinct. Parallax is produced by improper focus and creates the effect of unstable conditions in the position of the wires.

To test for parallax, after focusing the telescope on some fixed point, move the eye up or down, or from side to side, over the range of vision and see if the wire seems to leave the point. If so, it means that the images of the field and the cross wires are in two slightly separated focal planes. Depending upon whether the object seems to follow the movement of the eye, or move contrary, shows whether the image is beyond, or behind, the plane of the cross wires. In any event the objective will have to be slightly readjusted until the phenomenon is overcome.

As the work progresses through the day, one's eyes will become tired and a readjustment of the eyepiece will be necessary. We provide a wide range of focus in all of our oculars to fit all such inequalities of vision.

IV. To Secure a Position for the Cross Wires Normal to the Axis of Revolution.

The cross wires, in reality, are spider lines taken from the cocoon of the *Eperia Diadema.* They are mounted at right angles to each other and fastened to the diaphragm with shellac. This adjustment seeks to secure the proper position of one wire. If the vertical wire, for instance, is vertical, the horizontal wire will be horizontal. Proceed as follows:—

Select some point at convenient range and set the vertical wire upon it. With the lower plates clamped, move the telescope up and down with the tangent screw and note the result. If the wire adheres to the point, its position is correct, but if the point appears to pass from one side to the other, release the tension in all four diaphragm screws, and placing the blade of the screw driver against one of the washers tap gently in the direction desired and test as before.

It is not necessary that the plates should be leveled to perform this test, but following the process in test I it would be customary to have them so, and in this event the vertical wire may be tested against a plumb line.

This test may also be conducted by moving the instrument from side to side on the vertical axis and watching the contact of a certain point with the horizontal wire, as suggested for leveling instruments on p. 7.

V. Collimation.

A line of sight may be anywhere in the telescope. The stadia wires, for instance, mark two sight-lines that are considerably removed from the center of the field. A line of sight does not become a line of collimation until it occupies a very precise position in the optical axis.

Essentially, the collimation adjustment involves the fixation of the intersection of the cross wires in the optical axis of the objective. Incidentally it arranges a sight-line, or two intersecting planes of vision, that should not only be fixed at right angles with the horizontal axis in both planes but eventually, by the standard adjustment, must revolve in a plane that passes through the vertical axis.

Necessarily this requires that the adjustment should be conducted in two planes, although the position of the horizontal wire has been quite generally treated to culpable negligence. Some years ago Mr. S. P. Baird remarked * that "instrument makers throw the errors of eccentricity into the vertical wire of levels and into the horizontal wire of transits. The average transit", he concludes, "is, therefore, not intended for precise leveling".

One of our contemporaries announces that their instruments are "so mechanically perfect that the cross wires may be placed in the optical axis by simply placing them in the center of the field of view"; but we take issue with such impressions or assumptions. These are matters with which we need not temporize, because they are susceptible of examination and verification by the surveyor.

(a-1) Adjustment of the Vertical Wire.

The vertical wire marks a vertical plane of vision which must be made normal to the horizontal axis, or in plunging the telescope it will describe a cone, and straight lines may not be prolonged. In reading deflection angles there will also be a cumulative discrepancy due to collimation error. For reading horizontal angles, however, a constant error of this kind, applied to each observation, would not affect the net result if the focus of the telescope were not required to be changed. This adjustment has been quite generally called "the collimation adjustment", whereas in reality it is only an important half. We may otherwise describe the collimation line as a very particular element in the telescope whose termini are marked by the nodal point in the objective and the intersection of cross wires that have been placed in its focal plane and in its optical axis. Obviously, any movement of either of these terminal points in a direction that is not continuous with the line itself, must destroy that relationship and affect the accuracy of both the adjustment and the observation.

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* *Eng. News, Vol. LXV, 1901, p. 377.*

Proceed as follows:—Level the instrument; sight some well defined reference point with vertical axes clamped; plunge the telescope and make a mark as indicated by the vertical wire. If a lateral error exists, this reversion of the telescope will double it. Reverse the instrument on the vertical axis with telescope still inverted and sight at the test point a second time. Plunge the telescope again and mark the second position indicated by the vertical wire. If it coincides with the first point, the adjustment is correct. If not, bear in mind that the second reversal has doubled the error again in the opposite direction. The distance between the points is, therefore, four times the total error. Correct back one quarter of the distance between the test points. If the telescope is erecting, move the diaphragm to the side opposite to the one which seems to be necessary, but do not under any circumstances strain the adjusting screws. The test points may be on the ground, but theoretically they should both be in the horizon at distances each side of the instrument equal to to the average field sight. To make a trial of the accuracy of the draw-tube, however, first collimate on points 500 ft. distant then check the adjustment on a new set not over 25 ft. away. If the second test can not be reconciled with the first, either the focusing tube does not travel in the optical axis or the standards are not centered with respect to the vertical axis.

In our 4½-in. Tachymeter the erecting telescope is so arranged that the diaphragm and cross wires are set between the two lenses of the Huyghens ocular, and in this case the wires should be drawn in the direction apparently necessary. This will be true also for any inverting telescope.

(a-2) A second method of collimating the vertical wire may be accomplished without a helper, by assistance only of the graduated plates, on the assumption that the zero and the 180° lines are diametrically opposite.

Set Vernier A of the horizontal limb to zero and sight any object, or a well illuminated scale, at any convenient distance with both vertical and azimuth axes clamped. Plunge the telescope backward, open the vertical axis, revolve the instrument 180° and very carefully set the index of Vernier B to the zero of the limb. Now observe the relationship of the vertical wire with the original test point. If coincident, the test is satisfied. If there is a separation, move the cross wires back over half the distance.

(b-1) Adjustment of the Horizontal Wire.

If the transit is to be used for leveling, the same care in centering the horizontal wire should be exercised as in any leveling instrument. This process is facilitated in a Y-level by having collars at hand and wye bearings in which to invert the telescope. We have shown also how this test can be accomplished in a dumpy level, on p. 18, but neither of these mechanical conditions usually exists in the American Transit.

In 1909, we made for Prof. D. A. Molitor a transit in which the telescope was mounted revolvably in an outer axis sheath similar to the conditions that prevail in our telescopic alidades or in the J. B. Davis Solar Transit * or in the Arrol-Tancreed Theodolite ** but for quite a different purpose. This requirement would be better served, however, by adopting the ocular focusing device as prevalent in most European countries.

The correct position of the horizontal wire in the exact equator of the field of view is also important in the derivation of correct vertical angles, particularly in mining theodolites where vertical control is of paramount importance.

Proceed as follow:— Set a target, not less than 500 ft. distant, coincident with the horizontal wire. The junction can be made by slightly tipping the telescope, for in making this test it is not necessary that the telescope shall be in the horizon. The point sighted is only a reference point, and the effects of refraction need not be considered.

Turn the instrument, if necessary, a degree or two by use of the tangent screw and sight a scale, substantially placed, as near to the instrument as the shortest possible sight. To see the scale, such for instance as shown in Fig. 36, will necessitate running out the focusing system to the extreme limit by which an error in the placement of the horizontal wire, with repect to the optical axis will be registered but still intangible.

Revolve the instrument and invert the telescope. Carefully center the horizontal wire a second time on the distant target, or whatever the test point that may have been selected, and once more rack out the focusing adjustment to take a second reading on the scale. If the readings check, the adjustment is satisfied. If not, there is no well defined rule to follow in making the correction. Do not, however, correct back half of the error† but move the wire on, as if to increase the error, by an amount at least equal to the interval. Begin the test a second or a third time, and do not be dismayed if an error is still in evidence.

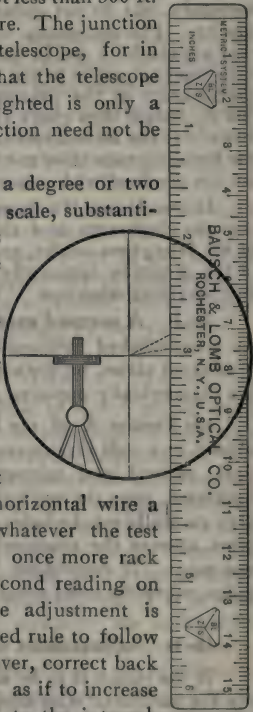

Fig. 36

* *Trans. Ohio Eng. Soc., 1895; also Jour. Asso. Eng. Soc., Nov., 1896.*
** *Trans. Inst. Mng. Engrs., Newcastle, Vol. 28, 1905, p. 642.*
†*J. A. Kitts in Eng. News, Nov. 26, 1914, p. 1080, is in error on this point.*

This method of test and correction may be likened to the process of ranging in a point on a hill with two flag poles in alignment with two points at opposite sides. The exact amount of the error cannot very well be measured, and the adjustment can only be accomplished by repeated test and correction until the line of sight assumes a stable condition for extreme positions in the field, in both normal and inverted positions. The principal precaution to observe is to guard against lost motion in the focusing rack.

(b-2) The Astronomical Method of collimating is the most rapid and accurate known. It requires a special ocular, suggested by Bohneberger in 1825, with an opening in the side between the lenses for the admission of artificial illumination. The telescope is focused for infinity and turned downward toward a vessel of mercury. As it approaches the nadir, a reflected image of the cross wires will be thrown back on the plane of the diaphragm. A coincidence cannot be made with the cross wires and their reflected image unless they lie in the optical axis of the objective, and this is just as true of the vertical as the horizontal wire.

After a test in one position the telescope should be reversed on the vertical axis to determine the error of horizontality in the horizontal axis which must be corrected. Ultimately when the cross wires are in the optical axis and directed to the absolute nadir, the instrument will spin on its vertical axis without any indicated deviation at the intersection of the cross wires.

This method is probably the best one known for the determination of true vertical angles and has been occasionally used for optical plumbing in mines in connection with specially constructed instruments with an enlarged and perforated vertical axis.

(c) The Adjustment of the Eyepiece is one which has no influence upon the accuracy of the operation. If the cross wires do not appear to be in the center of the field, when collimated as directed, it indicates merely that the field of the eyepiece is not concentric with the collimation axis. Some makers supply an extra set of adjusting screws for this purpose but our eyepieces are sufficiently centered in the process of construction.

VI. (a) The Adjustment of the Telescope Bubble.

This adjustment may be performed by any of the tests herein described for the dumpy level. A transit cannot be expected to perform even ordinary leveling unless the horizontal wire is properly collimated to the equator of the field; so that we may rationally conclude that any method prescribing a movement of the horizontal wire to agree with the axis of a bubble, casually applied, is essentially wrong. (Consult p. 92).

Having collimated the horizontal wire by the experimental test described under V, (b1), run the instruments over the pegs and in the final step move the bubble--not the diaphragm. The peculiar purpose of a set of pegs, as used for this method, is to establish an artificial horizon. If the collimated sight line is made to occupy a position in this horizon and the bubble checked up to agree with these conditions, the sight line will describe a level plane when the bubble occupies the center of its scale.

(b) If one has a reliable level at hand, a satisfactory adjustment of the telescope bubble may be accomplished by setting up both instruments so that the eyepieces are at the same height. Sight some distant point in the horizon through the level, bring the telescope of the transit to bear upon the same point, and adjust the bubble to this condition.

(c) Prof. S. D. Sarason of Syracuse University, in order to obviate the peg method, advocates the use of a reversion level to secure the adjustment for parallelism. The horizontal cross wire being collimated as directed under V, (b-1), proceed as follows:—

Level the telescope with the reversion bubble in a normal position by use of the tangent screw. Take a careful reading on a rod, say 300 ft. distant. Reverse the instrument in altitude and azimuth so that the telescope will be re-directed to the rod with the reversion bubble above. Center the bubble again and take a second reading. If the scales of the level vial register accurately, a mean position between the two observations on the rod will be a true horizon for that particular set-up. Bring the horizontal wire to read on this mean position by means of the vertical tangent screw, then, center the bubble with its own adjusting screws. Now reverse the instrument, and having set the telescope on the mean position a second time note the position of the bubble on its scale. If it is not perfectly centered, either the collimation adjustment for the horizontal wire is in error, or the tangential axes of the bubble scales are not parallel.

The Reversion Bubble (see p. 93) is a spirit level ground barrel shape on the inside surface so that the tangent axes of the opposite scales will be parallel. All manufacturers will acknowledge that this is a difficult task, and to get both bubble races ground to exactly the same curvature is even more difficult. We will not, however, send out a reversion bubble unless the scales register. A slight variation in the sensitiveness of the scales is of no consequence but the bubble must occupy the center, when the telescope is in the horizon, either erect or inverted. The bubble must, therefore, be adjusted to a properly collimated cross wire.

Otherwise it would be very easy to incorporate a constant error that might not be in evidence.

A movable metal scale* which the surveyor can adjust himself is obviously the best theoretical arrangement for the reverse side; but the use of such a scale precludes the use of a cover guard which is also important, and we are required, by preference, to graduate the reverse side of the vial after it has been applied to the instrument.

Fig. 37—Showing general equipment with Vertical Clamp, Telescope Bubble and Vertical Circle with Cover Guard and one Double Vernier, covered under the No. 045 4-in., No. 055 5-in. and No. 065 6-in. Transit.

VII. The Index of the Vertical Circle.

The index of the V. C. should read zero when the collimation axis lies in the horizon. Let us be certain of the premises in the case. The collimation axis is not a mere matter of chance, as we have shown in V, (b-1). When the horizontal wire is indeed collimated and the telescope bubble adjusted to parallelism with this sight line, the V. C., being permanently fixed to the telescope and revolving with it, should show no index error against its vernier scale.

*Invented by Col. Goulier in 1872.

The Vernier Scale is to be adjusted to the circle by moving it slightly in the groove provided, with the small capstan-head screws. The scale is fixed by the larger capstan screws which hold it to the guard in approximate position, as suggested in Fig. 37. Only the last minute or two are to be controlled by the smaller screws which operate against the side of the standards. Turn these in the same direction as in adjusting a diaphragm, first, by slightly loosening the one, then by tightening the other. If a large correction is to be made, the whole vernier frame is to be readjusted by loosening the tension in the screws at the back.

Opposite Verniers for the Vertical Circle must be built by the manufacturer so that the indices will be diametrically opposed, as in the horizontal limb. In this case they are fixed to a frame work which also forms a guard against accident to the reading edge. This arrangement of details makes it convenient to mount the guard on the same axle with the circle so that it will revolve in short arcs concentrically with it. Generally, the indices of opposite verniers are to be adjusted to the zero of the limb by capstan head studs operating against a nose-piece set into the standard leg nearest the ocular. In the Tachymeters and Theodolites of our manufacture we supply an **Index Adjuster** with a check nut which is to be operated, to accomplish this adjustment, like any other tangent screw. (See Fig. 38 p. 76).

VIII. **The Control Bubble**. See p. 94 .

IX. The Standard Adjustment.

This adjustment seeks to make the line of collimation revolve in a vertical plane that passes through the vertical axis. The collimation adjustment makes the sight line describe a plane instead of a cone and the standard adjustment brings that plane into a truly vertical position. It will not necessarily pass through the vertical axis, however, unless the maker has built a telescope whose optical axis is midway between bearings that are spaced equi-distant from the center.

The test points for collimation adjustment should properly occupy positions at extreme localities in the horizon, but the theoretically correct position for the test points in the standard adjustment is in the zenith and nadir. The object of the adjustment is to insure correct horizontal angles as read between points widely separated in elevation. The adjustment is of paramount importance in mining theodolites on this account. Recognizing this fact, in a recent work on this topic, the author has stood up so straight, however, that he has fallen over backward. The argument therein advanced, providing for a succession of taut wires and plumb lines, is not sustained or recommended.

The plate bubbles being in perfect adjustment, particularly the one lying transversally under the line of the telescope, and the vertical axis being truly vertical, proceed as follows:—

(a) Sight some fixed point as high above the instrument as can conveniently be observed. Depress the telescope and mark a point on the ground a short distance beyond the instrument. There is no need of going to the nadir. If an error exists, the amount would increase constantly with depth but the angular value would not change. Reverse the instrument in altitude and azimuth and re-center the telescope on the test point aloft. Depress the telescope a second time and compare with the first located point

If a coincidence occurs, the adjustment is complete. If not, recollect that the horizontal axis is at right angles to the direction of the last sight. If the last sight struck to the right, the right axis is too high and *vice versa*. A mean position between the two lower test points will be vertically below the upper one. Move the adjusting block up or down, as the case may be, until the vertical wire cuts the central point.

The adjusting block, in the upper end of the standard, is held in position between a capstan head adjusting screw beneath and the bearing cap above. It is always best therefore to bring the adjusting block into place by an upward movement of the capstan screw, but in case it has been moved upward a trifle too far, the excess can frequently be corrected by tightening the dowel screws in the bearing cap. This association of parts, however, ought not to be strained. Some skill and judgment, in fact, will be required to get the bearings nicely seated on each side without undue strain.

(b) **A second method** is a modification of the first. Bisect a well defined high point *a*; depress the telescope to the horizon, or ground, and mark a reference point, *b*, as indicated by the vertical wire. Reverse in altitude and azimuth and again sight the point, *a*; depress the telescope as before and locate a third point, *c*. By means of the slow motion screw, turn the alidade until the vertical wire is midway between *b* and *c* then elevate the telescope on this alignment to the locality of the point *a*. This upward movement will describe a path parallel to the last downward plunge. Therefore correct the whole error indicated in order to get the cross wires vertically over the lower median point.

When in 1886, A. V. Lane was Ass't Prof. of Math. in the Univ. of Texas, he published a *Handbook of Adjustments* in which he successfully proved, by descriptive geometry, that the correction point was not necessarily midway between the two test points. This abstruse principle, however, has not been generally noticed, for adjustments in nearly all cases are matters of trial and no one will be satisfied with a test until it has been verified by repetition.

In projecting alignments in steep incline shafts the process of this adjustment should be put into practice, so far as possible, on every sight. If mine waters are acidic and corrode the axles, one can never be certain that this important adjustment has not been modified. The mechanical treatment of the bearing itself is a matter of prime importance in the permanency of adjustment.

(c) The easiest and most accurate method for the standard adjustment, and the one that provides the best control for triangulation, or tunnel alignment, is unfortunately not much used because of the exposed position of the **Striding Level** which is employed. This method of adjustment is described on p. 131.

Bausch & Lomb 4-inch Transit

"Reconnoissance"

Specifications

Telescope—Erecting, 8¼ in. extending to 9 in. long; aperture, 1⅛-in.; power, ×18. Inverting, ×16; transits at eye-end.

Telescope Bubble—3¼ in. long, 2mm divisions, sensibility, 40″.

Plate Bubbles—1½ in. long, 2mm divisions, sensibility, 60″.

Compass Needle—2⅞ in. long, tubular, sapphire mount, weight, 0.47 grms.

Vertical Limb—4-in. diam., minute graduations, adjustable vernier.

Horizontal Limb—2⅛-in. rad.; 4¾-in. diam. outside graduations.

Standards—Bronze, 4⁷⁄₁₆ in. high, 2¼-in. base.

Leveling Screws—German silver, ⅞-in. head, dust caps.

Shifting Center—½-in. range.

Height—9 in. with arc; 10 in. with circle.

Finish—Gamboge yellow and Japanese black.

Weight—6½ lbs.; in case with accessories, 11½ lbs.
Split leg tripod, 57-in. legs, 7¼ lbs. Ext. tripod, 7½ lbs.

Bausch & Lomb 5-inch Transit

"Surveyors"

Specifications

Telescope—Erecting, 10 in. extending to 11¼ in. long; aperture, 1¼ in.; power, ×20. Inverting, ×20 or ×28, transits at eye end only.

Telescope Bubble—4¼ in. long, 2mm divisions, sensibility, 30″.

Plate Bubbles—2 in. long, 2mm divisions, sensibility, 50″.

Compass Needle—3⅞ in. long, tubular, sapphire mount, weight, 0.64 grms.

Vertical Limb—5-in. diam., minute graduations, adjustable vernier.
Horizontal Limb—2¾-in. rad.; 6¼ in. diam. outside graduations.
Standards—Bronze, 5⅝ in. high, 2⅞-in. base.
Leveling Screws—German silver, 1¼-in. head; dust caps.
Shifting Center—⅝-in. range.

Height—11¾ in. with arc; 13 in. with circle.
Finish—Gamboge yellow and Japanese black.
Weight—12¾ lbs.; in case with accessories, 23 lbs.
Split leg tripod, 60-in. legs, 9½ lbs. Can be reduced for municipal work when requested. Ext. leg tripod, 10¾ lbs.

Bausch & Lomb 6-inch Transit

"Municipal"

Specifications

Telescope—Erecting, 11¼ in. extending to 13½ in. long; aperture, 1½ in.; power, ×22.5. Inverting, ×14, ×24 or ×34; 12¼ to 13½ in. long; transits at eye end only.
Telescope Bubble—4⅝ in. long, 2mm divisions, sensibility, 25″.
Plate Bubbles—2 in. long, 2mm divisions, sensibility, 50″.
Compass Needle—4½ in. long, tubular, sapphire mount, weight, 0.72 grms.
Vertical Limb—5-in. diam., minute graduations, adjustable vernier.
Horizontal Limb—3-in. rad.; 6¾-in. diam. outside graduations.
Standards—Bronze, 6⅞ in. high; 4-in. base.
Leveling Screws—German silver, 1¼-in. head; dust caps.
Shifting Center—9/16-in. range.
Height—13⅛ in. with arc; 14½ in. with circle.
Finish—Gamboge yellow and Japanese black.
Weight—18 lbs; in case with accessories, 31 lbs.
Split leg tripod, 60-in. legs, 11½ lbs.; Ext. tripod, 13 lbs.

Fig. 38—Showing general equipment with Vertical Clamp, Telescope Bubble, Vertical Circle with Open Cover Guard (enclosed if desired) and Double Opposite Verniers; covered under the No. 46 4½-in., No. 56 5-in. and the No. 66 6-in. Tachymeter.

Note:—The new 3-screw base, gradienter, control bubble, fixed stadia, etc., are among the accessories furnished only on request.

THE TACHYMETER

TACHYMETERS and Theodolites of our design have incorporated in their structure all the refinements of manufacture known to the instrument maker's art. The supreme technical skill of the artisan, as well as the quality of the material and perfection of the equipment, are the controlling factors in design and production. Each instrument is sustained by the conventional guarantee that covers the entire business relationship of this house.

The best materials only are employed. The inner center is bell metal; the intermediate center, phosphor bronze; the outer center, hard red composition and the leveling screws, German silver. The standards are cast in one piece of aluminum bronze, and the journals rotate on cylindrical bearings, of ½-in. diameter and ⅜-in. width, in morticed blocks of phosphor bronze.

The Axis Bearings are true cylinders of equal diameter—universally acknowledged to be the ideal construction for any transit instrument. Bearings that are recessed are weakened, and the scheme multiplies the number of surfaces that are to be kept in nice relationship for the important purpose of the horizontal axis adjustment.

The recessed bearing is intended only to overcome side play in the telescope journal, but we accomplish this bond by an entirely different means which does not interrupt a continuous bearing surface. The bearings of all transit instruments that are used in flat country, where the telescope is tipped only a little above or below the horizon, will sooner or later get a little worn on the under side. The cylindrical axis bearing will unquestionably last the longest, but it is always good practice to equalize wear by using the telescope a part of the time in an inverted position. In triangulation this recourse is also necessary to correct for the eccentricities of a collimated telescope with respect to the center of the horizontal circle, (see p. 129). A telescope that will not qualify as to collimation adjustment for all distances is either mounted to one side of the center or there exists an imperfect motion in the focusing slide. B. & L. bearings are spaced equidistant from the center by a very accurate mechanical process, after which the telescope is permanently fitted, concentrically, without adjusting screws for the focusing tube, so that the collimation test is satisfied for all distances.

The Finish is a new weather-proof material in an olive drab (khaki) tint which is very agreeable to the eyes in all climates. This has superseded the cloth finish, which is used to cover a multitude of sins and cannot be cleaned.

The Vernier Windows are set, according to prevailing custom, at about 30° off the direction of the sight line. This plan makes the observation of Ver. A. a very convenient undertaking

(see top p. 103), and where Ver. B. is consulted, the effort is no greater than when opposite verniers where placed between the legs of the standard.

These instruments are designed so that the telescope swings through the standards at the objective end without special preparation or precaution (see p. 189) and, while the sun shade may occasionally touch the frame of the vernier reflector, there will be no danger of damage incident to the contact. Special provision in the way of buffers are not necssary with these new telescopes.

Occasionally for mining purposes we have placed the vernier windows immediately under the line of the telescope. In cramped positions, this arrangement is very convenient.

The **Edge Graduated Vertical Circle**, for the same reason, is especially important. In boggy ground or in other precarious positions the instrument that permits readings to be taken without moving in one's tracks offers special advantages that will be appreciated by those who have experienced the discomforts and irritation of an inaccessible vernier opening.

The Lens System

The interior focusing system, peculiar to the Tachymeters and Theodolites, and the variable power eyepiece, applicable to all classes of instruments, are distinctive features which merit detailed description and explanation. Before entering upon their description, however, we present a brief exposition of the theory of the telescope which we believe will be of interest to every engineer.

The Function of the Telescope is twofold: first, it serves to bring the object into the plane of the cross wires without parallax and second, it serves to make visible objects which would be invisible to the naked eye. The extent to which the first of these is accomplished depends entirely upon the accuracy with which the telescope is focused. The performance of the telescope with respect to the second consideration depends upon the resolving power of the objective, magnification, brightness of image, and the perfection attained in the design and manufacture of the lenses.

The Resolving Power of an objective is the measure of its ability to form separate and distinct images of two neighboring points of the object. Its value depends upon the aperture of the objective and is expressed by the angle subtended at the center of the lens by the distance separating the two closest points which it can image separately, i. e., resolve. The relation connecting resolving power and aperture is $\theta = \frac{12.6}{D}$ wherein θ is the resolving power in seconds of arc, and D is the aperture expressed in centimeters. It applies without modification only to white points on a black back ground. An objective of one inch aperture will then

resolve two points which, from the center of the objective, appear under an angle of 5″. An objective of two inches aperture will resolve 2½″.

The importance of resolving power cannot be too much emphasized. To make the matter more concrete, an angle of 5″ will include about 15.3 in. at a distance of one mile. Two points separated by 15.3 in. will just be resolved by an objective of 1 in. aperture. They will be visible as two points if the illumination of the object and the magnification of the telescope are sufficient. If the aperture of the objective is less than 1 in. no amount of magnification nor of light will suffice to dispel the appearance of the image as a single point.

As has been intimated, resolving power alone is not sufficient to make the two points visible as such through the telescope. Before two points can be seen as separate they must be made to appear under an angle of at least 1′. This makes necessary a certain degree of

Magnification in the telescope. The degree of magnification necessary is determined by the resolving power of the objective. The power to resolve 5″ requires a magnification of at least ×12. Without this magnification, two points separated by 5″ will be imaged separately in the focal plane of the objective but they will not be separately visible because the angle under which they would appear to the eye through the eyepiece would be less than the necessary minimum of 1′. Magnification exceeding this amount, while it would increase the angle under which the two points would appear, could not render visible a third point lying between the two. The explanation lies in the fact that the objective cannot image a point as a point, but only as a disc of finite size surrounded by alternate concentric bright and dark rings. The larger the objective the smaller the diameter of the disc, hence the greater the resolving power. Magnification exceeding the necessary minimum, while increasing the angle subtended by the centers of the disc images, increases the apparent size of the discs at the same time thereby preventing detection of any intermediate detail.

Excessive magnification in addition to being useless is positively detrimental under any but the best of light conditions.

The Brightness of the Image of a telescope is inversely proportional to the second power of the magnification and directly proportional to the second power of the aperture. The expression for the absolute illumination of the image is, $I = A\frac{d^2}{m^2}$, where A is a constant when comparing two instruments side by side on the same object, **d** is the aperture of the objective, and **m** the magnification. A 10% increase of aperture will give over 20% increase in brightness while 10% increase in magnification will result in about 17% loss in illumination.

By looking at the eyepiece of a telescope with the head held back a few inches a small round disc of light will be seen. This is an image of the objective formed by the eyepiece. It is called the exit pupil for the reason that all the light which emerges from the telescope must pass through this circle. If we use **d** and **m** in the same sense as above, the diameter of the exit pupil is equal to $\frac{d}{m}$. Since the brightness of the image was found to be proportional to $\frac{d^2}{m^2}$ it is proportional to the square of the diameter of the exit pupil.

The apparent illumination of the image seen through the telescope is dependent upon the relation between the diameter of the exit pupil and the diameter of the pupil of the eye. Maximum brightness is obtained only when they are equal. If the exit pupil is smaller in diameter than the pupil of the eye, the brightness is less than the maximum; if larger, the brightness remains at the maximum value, for not all the aperture of the objective will be utilized. The diameter of the effective aperture, in the latter case, will be equal to the product of the magnification and the diameter of the pupil of the eye.

Quality of Lenses. The performance of a telescope can not come up to the measure of the theoretical value in respect to resolving power or illumination if the lenses are defective in either design or technique. Surfaces which are not perfectly polished cause the image to appear grey, surfaces which are not spherical result in images which appear drawn or strained when the objective is racked in and out of focus. If the lenses, especially the objective, are not centered the image will come into and go out of focus with a one-sided lack of symmetry. Technical skill of highest order and the best of mechanical equipment are necessary for the production of high grade telescope optics but these will be of no avail if the design of the lenses is faulty.

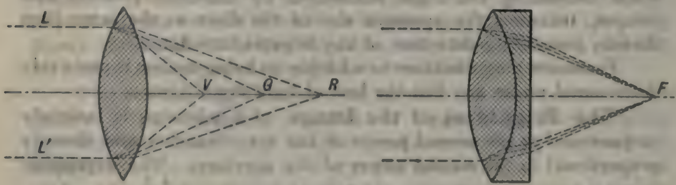

Fig. 39

A single lens used as a telescope objective has two principal defects: chromatic and spherical aberrations. The focal length of any single lens is different for each different color of light so a beam of white light, composed as it is of a mixture of all the colors of the spectrum, instead of uniting in a single focal point is distributed

along the axis in a series of focal points as in Fig. 39. This is chromatic aberration. This aberration is of opposite sign in positive and negative lenses so that the combination of a positive and a negative lens of the same power and same kind of glass will be free from chromatic aberration but it will have no power. Fortunately, glasses of widely different dispersive powers are available so that we can make a negative lens of relatively low power which has chromatic aberration sufficient to neutralize that of a relatively strong positive lens. The combination will then be achromatic but will have a residual positive power equal to the algebraic sum of the powers of the elements.

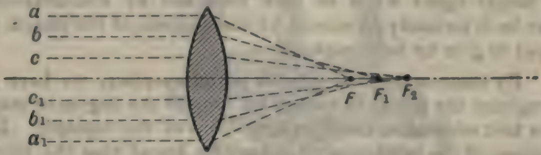

Fig. 40

Light incident on the marginal zones of a lens is refracted relatively more strongly than rays incident near the axis, so that, even if we restrict ourselves to light of a single color, the incident beam is brought, not to a single focal point, but to a series of focal points as shown in Fig. 40. The magnitude of this so called spherical aberration varies with the shape of the lens and is a minimum (not zero) when the work of refraction is equally divided between the two surfaces. The spherical aberration of a negative lens is of such a sign as to tend to neutralize that of a positive lens. Hence, after the focal lengths and glasses have been selected in such a manner as to eliminate the chromatic aberration, the positive and negative elements of the achromatic lens may be so shaped that their spherical aberrations are equal in amount and therefore neutralize. The combination is then free from primary chromatic and spherical aberrations.

It must not be assumed, however, that a so-called achromatic lens will produce an absolutely colorless image. The ratio of the dispersions of the crown and flint glasses available for telescope objectives are not equal for all parts of the spectrum so it is possible to bring to a common focus only two colors. This leaves some uncorrected chromatic aberration known as secondary spectrum. Further, chromatic correction can be effected for but one zone of the objective. Corrected for the axial zone an objective will show over correction at the margin, a defect which becomes rapidly more serious as the relative aperture is increased. Up to a certain point, however, we believe the increase in brightness gained by increase in aperture more than counterbalances the increase of color in the image.

The Variable Power Eyepiece, supplied solely by this house, is an accessory whose purpose is to add flexibility to the telescope. The ordinary surveyor's telescope is provided with a single eyepiece which gives a degree of magnification which must be a compromise between that value which enables the longest sight to be taken and the value which affords the brightest image. Even if this compromising value be intelligently chosen, the engineer is losing some of the possibilities of his instrument. On bright days he could use still higher power and take longer sights while on dark days and by mid-afternoon in winter he is seriously hampered by the dullness of the image, if not obliged to give up field work.

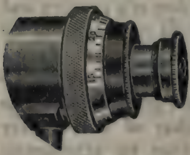

Fig. 41

By constructing the variable power eyepiece, we have given the engineer the power to choose for himself the degree of magnification best adapted to the light conditions of the moment. In the discussion of the brightness of image, we found that the illumination was inversely proportional to the square of the magnification. Our variable power eyepieces have a range of magnification such that the ratio of the lowest to the highest power is in some cases 1 : 2 and in other 3 : 5. In the first case, then, we give the engineer the power to vary the illumination in the ratio of 4 : 1 while in the second, the range is somewhat less than 3 : 1. This means a substantial increase in the length of the engineer's working day in the field. To the gain in this respect must be added another gain made possible by the use of the low power. When the air is boiling under a hot sun the ranging rod often appears so unsteady that readings are impossible. The use of the lowest power will materially reduce the trouble and enable work to be done which without the variable power eyepiece would have to be postponed.

The principle involved in the change of magnification is identical with that employed in the old pancratic eyepiece. The eyepiece as a whole is of the erecting type and consists of an ocular and an erecting system, and ahead of these, mounted absolutely independent of the eyepiece, is the cross wire reticule. By altering the distance between erecting system and cross wires and then moving the ocular alone until the cross wires are again in focus, the focal length of the eyepiece, and therefore the power of the telescope, has been changed. The cross wires in no way participate in this movement but remain in fixed position attached to the telescope tube and collimation is no more affected by this operation than by leveling the instrument.

To operate the attachment, first set the knurled ring to the magnification desired and then focus the ocular so that the cross wires are brought into focus. The power may be changed, while

Fig. 42—Showing general equipment with Vertical Clamp, Telescope Bubble and uncovered Vertical Circle, as supplied with the No. 44 4½-in., No. 54 5-in. and the No. 64 6-in. Tachymeter.

the telescope is focused on some particular object, without touching the objective, for the image formed by the objective lies in the plane of the cross wires and when the cross wires reappear in sharp focus the image will also be in focus.

We cannot furnish the variable power eyepiece with a 4-in. or 4½-in. instrument. With the 5-in. instrument a range of power between ×15 and ×25 is possible; with the 6-in., between ×17 and ×27; with the 16-in. wye level, between ×13 and ×26 and with the 21-in. wye level, between ×18 and ×36. In the last two cases the cross wires reappear as the magnification is doubled. We cannot supply the variable power with the inverting telescope except by furnishing extra oculars of various focal lengths which are to be interchanged as the conditions require.

The New Interior Focusing System is peculiar to our Tachymeters and portable Theodolities. It is the result of an effort to make these instruments as nearly perfect as possible. Having given our best attention to the quality of the lenses, the accuracy of the graduations, and the rigidity and accuracy of the mechanical construction, the principal outstanding source of error was the disturbance of collimation adjustment in the process of

focusing the telescope. No matter how well fitted the objective or ocular draw tube may be when they leave the manufacturers hands the inevitable wear soon permits lateral displacement with its well known effect on the collimation. We were therefore forced to abandon both of the older types of focusing which required movement of either the objective or of the cross wires, and to seek some means of focusing which would permit these two elements to remain in unalterable relationship with respect to each other.

Fig. 43

This condition required the interposition of some moveable element between the objective and cross wires and at first glance it might seem that lateral displacement could not be avoided here any more than in the older types of focusing adjustment. In fact it cannot although it can be reduced in amount and the feature of most importance, the effect of a given amount of displacement, has much less effect on collimation than an equal displacement of objective or cross wires.

We found that the conditions were best fullfilled by employing a negative lens of low power as the focusing element. The manner of its working is shown in Fig. 43. The cross-wire diaphragm at F′ is placed beyond the principal focus of the objective at F. The plane of the cross wires is then conjugate to an object plane at a short distance ahead of the objective and if the negative lens is brought into the plane of the cross wires so that its effect is zero, that near object will be in focus. An object at a great distance will be imaged by the objective in its focal plane at F, but by shifting the negative lens towards the objective a point will be reached where its dispersive action is sufficient to prevent the formation of an image ahead of F′ and therefore this image also lies in the plane of the cross wires. Between these limits an object at any distance may be focused on the plane of the cross wires by moving the negative lens to the necessary position.

Fig. 44

The negative lens must, of course, move back and forth with a minimum of lateral displacement. To this end the focusing lens barrel is guided by three longitudinal ribs integral with the telescope tube and turned to a center in common with the objective mount. Fig. 44 shows a cross section of the tube. This arrangement provides an accurate seating for the focusing-lens barrel throughout its entire range.

Assuming, however, that slight lateral displacements occur we can demonstrate that the effect on collimation is much less than the effect of an equal displacement of the objective. Referring to Fig. 45, let O represent the objective and R the diaphragm. Let N be the position of the negative lens when the collimation line, AB, was established and assume that in focusing on a nearby object the negative lens moves out of alignment to the position N_1. F is the

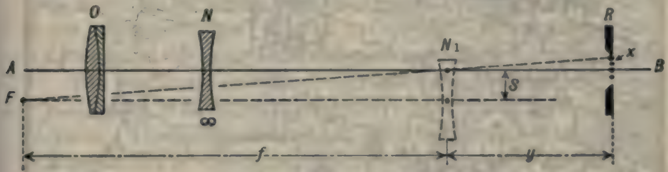

Fig. 45

anterior focal point of the negative lens and f its focal length. The axial ray through the center of the objective, O, will be deviated by the focusing lens as shown in the diagram and produce a deflection x at the diaphragm. If s represents the lateral displacement of the lens we have: $x : s :: y : f$

$$x = \frac{sy}{f}$$

In our 5″ Erecting Tachymeter $f=285$ mm and $y=92$ mm when focused on a point 5 ft. ahead of the objective. Let us assume $s = 0.02$ mm, then

$$x = \frac{0.02 \times 92}{285} = 0.0064 \text{ mm or about } \tfrac{1}{3}\, s$$

In our 5″ Inverting Tachymeter $f=448$ mm and $y=148$ mm when focused for 10 ft. Substituting as before,

$$x = \frac{0.02 \times 148}{448} = 0.0065 \text{ or again } \tfrac{1}{3}\, s$$

Whatever the lateral displacement of the focusing lens the effect upon collimation does not amount to more than the third part of it. In the older types of focusing any lateral displacement has its full effect upon collimation.

In addition, the interior focusing system permits more nearly water and dust proof construction as well as less disturbance of balance in the process of focusing.

Fig. 46—Lines of Equal Magnetic Variation in the United States
(From U. S. Forest Service Manual)

We have specialized in magnetic investigations only as an incident in the manufacture of the more accurate instruments in which scientific optics and precise mechanics control the issue.

The discovery of the north-seeking needle is traceable back to rather remote antiquity among the Chinese, who observed this specific property in polarized steel but were never able to apply the phenomenon to the science of terrestrial magnetism which is now of equal importance, in the art of navigation, with the needle itself. As a means of rapid approximate orientation a more serviceable instrument has never been devised, but as a means of conducting land or mining surveys it must be regarded as belonging distinctly to the past generations. The U. S. Gov't. will not permit mining claims or new township locations to be surveyed with the compass.

The needle was introduced in Europe about the year 1300, but the natural laws affecting its behavior were not studied until 1541 in Paris. The needle declines to remain stationary for any length of time on any of the habitable portions of the earth's surface. In London, for instance, it changed its direction 24° 28′ in the 155 years preceding the war of 1812. Shortly afterwards it began to swing back again and has since changed its position more than 10°. The change is not so rapid in the U. S. as in Europe, but it is nevertheless sufficient to cause a serious error in surveys conducted with it, if its rate of change were not known. A boundary line of a mile, fixed with the needle in Maryland in 1802, would now have the one end more than 500 ft. away from its original location if retraced by this same method. Again, the mean declination, for instance, in the State of Illinois for 1870 was 6° 15′ E. By 1900 the average declination was 4° E., showing that in 30 years it had changed at the rate of 4′ 30″ per annum.

Points on the surface of the earth showing equal magnetic variation may have drawn through them irregularly curved lines known as isogonic lines. The lines of no declination are known as the agonic lines. In 1855 the Coast Survey published its first isogonic chart. The one reproduced in Fig. 46 is taken from the *Forest Service Manual*. The lines will have to be replotted in the course of time, for the entire system is slowly changing its position. The American agonic line passes through the U. S. near Charleston, S. C., Cincinnati, O., and Lansing, Mich. The European agonic line passed eastward through London in 1657, Paris in 1669 and is now in the vicinity of Petrograd, as shown in Fig. 47.

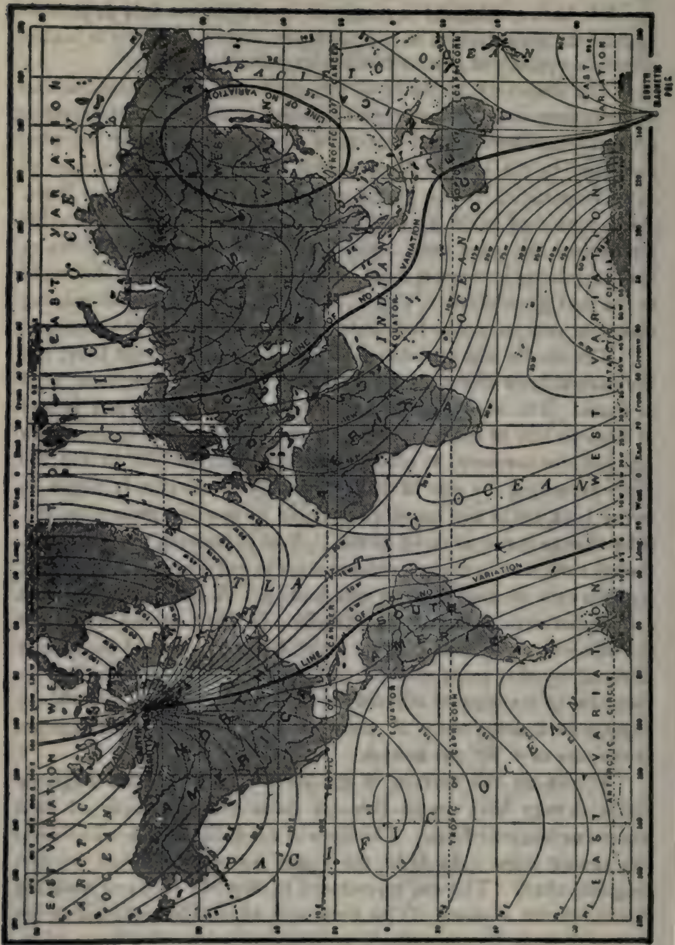

Fig. 47—Isogonic Chart of the World on Mercator Projection
(From Pence and Ketchum Manual)

The agonic lines on the surface of the earth are in reality the positions where the influences of terrestrial magnetism and local attraction are equalized. The earth is a great magnetic field having its north magnetic pole on the western shore of Boothia Felix in Canada and its southern magnetic pole in that great south polar region recently explored by Amundsen, Shackleton and Scott.

This directive force of the earth's magnetic currents is divided into horizontal and vertical components, the first being measured by the "Magnetometer" and the second by the "Dip Needle". The first manifestation is what produces declination—commonly known by surveyors as the variation of the needle—and the second produces the unequal balance that must be compensated in the northern hemisphere by slipping a little movable weight over the S. end of the needle. This identification will always distinguish the South from the North end of the needle. At the equator the magnetic forces play horizontally over the surface of the earth and the needle requires no artificial balance, but in the southern hemisphere the counterweight must be transferred to the north end.

A needle balanced for the latitude of Rochester will not necessarily float freely elsewhere. To readjust this condition simply unscrew the compass glass and, removing the needle, keep sliding the counterweight along until, by repeated trial, a perfect balance is secured. Wipe the needle free from finger stains on the final replacement. Our method of mounting the compass glass makes the needle easily accessible but keeps it free from moisture.

Our needles are made of tubular tungsten steel of the lightest possible weight, averaging about 0.165 grms. per inch (see p. 127). The unusual superficial area permits the highest limits of magnetic saturation and the lightness reduces friction on the pivot, making the needle very responsive. An accurate needle is scarcely ever in absolute repose, for the pen points at the ends and the point of support should be as nearly as possible in the same straight line.

Magnetic bearings are read on an extended traverse as a check against such errors as arise from using the wrong tangent screw or in making an incorrect reading on the vernier; but to guard against the effects of local attraction, magnetic records of both forward and back-sights should be taken and corrections applied. In this event, however, nothing is gained over doubling every observation on the vernier plates to prove the first reading by seeing that the second is double its value.

The sum of the interior angles of a polygon, as deduced from magnetic observations, will not necessarily represent the theoretical sum; for whether right or wrong, the aggregate will apparently prove the work. Such angles, if read to the nearest 15′, will often check up by compensation, or chance, while the same angles carefully determined on the graduated plate will fail to check.

The Continuous Variation Plate

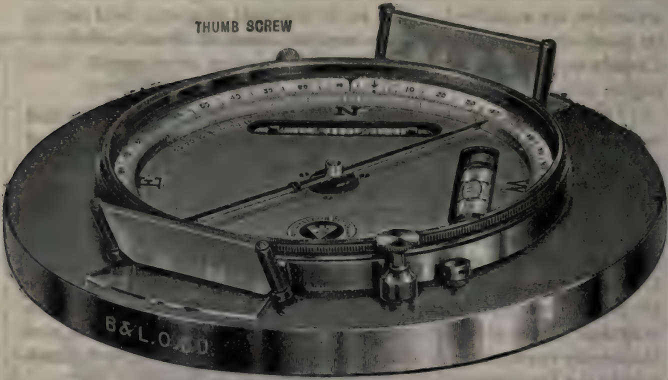

Fig. 48—Continuous Variation Plate, Showing Inclined Reading Surface, Vertical Graduations and Vernier Scale. Illustration shows circle set for a variation of 7° E.

The magnetic declination at any place is the angle contained between two vertical planes, one being the true meridian and the other the plane in which the axis of a freely suspended needle will lie, as an algebraic result of the various periodic and irregular influences that control the direction of the needle. Among these may be briefly mentioned:—

Terrestrial Magnetism, which is the principal directive force induced by the currents that pass between the earth's magnetic poles.

Secular Variation, which shows charted oscillations as great as 24° in one direction—spread, however, over a cycle of years that has not yet been closed since magnetic observations have been a matter of record.

Annual Deviation, which changes mainly with the seasons, being more conspicuous in summer than in winter, varying roughly between 5′ and 15′ per year.

Diurnal Variation, being a systematic angular movement of from 5′ to 10′ covering a solar day, doubtless due to the attraction of the sun.

Local Attraction, which is very noticeable in the vicinity of slate, pyrrhotite, magnetite deposits, or industrial or electric plants. It not only affects the horizontal position of the needle but frequently interferes with the vertical balance.

Lunar Inequalities, Solar Phenomena and Magnetic Storms, which often exert sudden deflections that are beyond the power of prediction. Magnetic disturbances accompanied by

auroral manifestations have been known to deflect the needle more than 20°.

The magnetic needle freely suspended on a silk fiber is known, as a matter of observation, to be seldom or never at rest. The needle which requires the longest time to settle will doubtless be he most trustworthy in indicating the algebraic sum of the magnetic forces at the place of observation.

This total magnetic force, pulling the needle either to the right or to left of the true meridian, is known as the "Declination of the Needle". Surveyors and mariners call it "variation" so that the scale upon which a correction may be made for this error is known as the Variation Plate, in contradistinction to the Declination Arc which is used in connection with solar attachments for quite a different purpose. In the spring of 1913 we designed a new variation ring, which not only compensates for horizontal displacement but for about 3° of dip as well. This construction permits us to turn off by vernier, to the nearest minute, any variation up to 90°. This recourse has been devised to meet the personal preferences of certain surveyors who are particular about this idea. If the needle is not properly balanced, or the vertical component of the earth's magnetism suddenly becomes excessive, the needle will dip downward, or a wind storm may so charge the compass glass with static electricity that the needle will rise and occasionally adhere to the glass. The first case will be met by the graduations on the inner vertical edge of the ring, or perhaps by readjusting the counterpoise weight if the conditions grow severe; but the second annoyance must be dispelled by breathing a coat of moisture over the glass cover.

To Turn off Variation in any required amount, loosen the thumb screw in the compass rim, as shown in Fig. 48, and push the circle along past the index that is just above the letter "N". If the ordinary limitations are not sufficient, remove the thumb screw and insert it in the next hole. Variation to the nearest 10′ can be set by inspection, but if one wishes to exceed this limit of accuracy, the rest must be done by vernier. We never furnish the variation vernier except on special order.

If one attempts to read a needle to the nearest 10′, the uncertainty of reading, with other errors, is perhaps 5′ or 6′, which would cause a probable total error of 10′ or more for each observation and an error of closure of at least double this amount.

(See also Declinatoire page 140.)

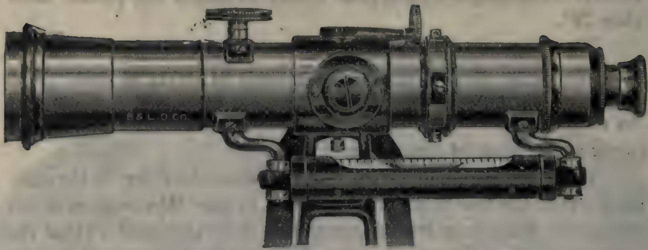

Fig. 49

Generally speaking, the scientific aspects of the Telescope Bubble and the functions assigned to it have been treated with scant consideration. The question that may rightfully be considered is:—Shall the bubble axis be adjusted to the line of sight or the line of sight to the bubble axis? One of these premises must be eminently right, the other emphatically wrong. There is no optional choice, as has been so freely allowed by manufacturers, educators and practitioners.

One would not attempt to adjust a bubble to parallelism with one of the stadia wires and undertake to do leveling with this combination of expedients. In order that we shall be favored with orthoscopic vision at all distances, the horizontal sight-plane must be collimated to the optical axis. The sight line must therefore lie at the basis of adjustments, and the telescope bubble must be provided with mechanical arrangements by which it can be adjusted to the horizontal collimation plane without strain.

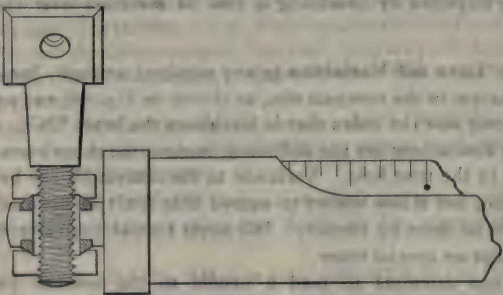

Fig. 50

When the vial mount is provided with adjusting facilities at only one end, strain is not only likely but very probable. The fixed shoulder at the end nearest the object glass is an unyielding argument against high class construction. The inserted cut, Fig. 50,

will show how our vial mount is guarded against compound flexure by the spherical washers and open bosses.

The vial itself is held in position by spring tongues only, and sealed at the ends with cork in preference to plaster-of-paris, which offers resistance to ordinary expansion and interferes with field repairs.

Wherever the construction of the instrument is such that the **extension brackets** are required to facilitate reading the bubble, we are in the habit of substituting them, as indicated in Fig. 49. This is especially necessary where the vertical limb is covered with a closed guard. Our vertical arcs are designed to permit an unobstructed view of the bubble between the legs of the standard, as indicated in Fig. 42.

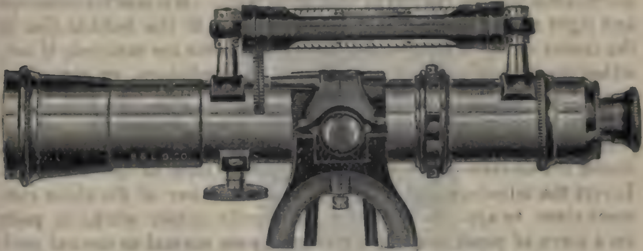

Fig. 51

There are two or three excellent reasons why the **Reversion Bubble** should be employed when the transit is much used for leveling purposes: 1st, the visibility; 2nd, the beneficial effects upon the telescope axis, and 3rd, the availability for rapid parallelism tests, as described on p. 70. The extension brackets touched upon above improve conditions without perfecting them and add nothing to the artistic mechanical treatment of the question.

If the circle is entirely covered, as usually prefered for mining or tunnel surveys, or for work in humid climates, the bubble cannot be seen from the left side of the instrument unless at the top of the telescope. A rather desperate alternative in such a case is to adjust the ordinary bubble upside-down so that it may not be used at all except at the top. With a control bubble to regulate the vertical circle, as shown in Fig. 38, such a plan is not without commendation.

In such flat country as skirts the Gulf sea board we find engineers taking all observations between the limits of, say, 5° above and 15° below the horizon with the result that in the course of time the telescope axis becomes worn and flatened in this segment, mostly for lack of oil. If the surveyor would use his telescope reversed in altitude for half of his work, he would at least double the life of his telescope axis, and a reversion bubble would contribute materially to this plan.

If manufacturers were frank with their clientele, they would acknowledge the difficulty of producing reversion bubbles that are ground to the same sensitiveness on both sides. To get the opposite scales to register perfectly also requires ingenuity, and the reversion bubble is worthless as a means for testing collimation (see p. 70) or collar inequalities in wye levels (see p. 14) unless they do.

We had better reverse the premises. Instead of testing the horizontal collimation with the reversion bubble which, however, is not impossible if the maker is faithful to his obligations, let us collimate as described under V, b-1, p. 68, and **test the reversion bubble** as follows:—

Level the telescope by the obverse scale and note the particular point in the horizon cut by the horizontal wire. Reverse the telescope and sight the horizon point a second time. If the bubble lies in the center of the reverse scale, the requirements are satisfied; if not, either the bubble axes are not adjusted to parallelism, or the opposite scales do not register. Run over the peg test and prove the first condition and finally test both scales against the horizon line so established. If both scales will not check up against this test, center the obverse scale with the adjusting nuts of the vial tube. Invert the telescope and either note the number of divisions constant error to apply with the proper sign in future work, or paste on a piece of paper and mark thereon a new normal or central position for the bubble.

Our reversion bubbles are guaranteed. Whether both sides are of the same sensitiveness is of no serious concern, but the tangential axes of both scales must be parallel. They are covered with a revolvable protection guard, not shown in Fig. 51, that is painted white on the inside to use as a background for either scale.

The Control Bubble

The Control Bubble performs an office similar to that of the Latitude Level but for quite a different purpose. When the horizontal wire is properly collimated and the telescope bubble adjusted to this sight line, and centered, the index lines of the vertical circle and its verniers must coincide. These various steps are taken up in detail on pp. 68 to 72.

Whereas the plate bubble lying across the longest dimension of the telescope is most important in preserving ideal conditions in the horizontal axis, the plate bubble running parallel with the telescope is the one that is of greatest importance in the matter of correct vertical angles. If the horizontal plates are not exactly horizontal in the direction of the line of sight, no appreciable error will be observable in the horizontal angle, but the vertical angle read will be in doubt.

Fig. 52—Showing general equipment with Vertical Clamp, Telescope Bubble, Vertical Circle with Cover Guard (open or closed, as desired) and one Double Vernier at the top, as covered under the No. 45 4½-in., No. 55 5-in. or No. 65 6-in. Tachymeter

To overcome the effects of temporal displacements in the horizontal plates, the vernier, or verniers, of the vertical limb are attached to the cover guard and mounted so that they may move in short arcs concentrically with the vertical circle. On the cover guard is mounted a level vial, as shown in Figs. 38 and 71, which is brought to the center of its run when the requirements expressed in the first paragraph above are fulfilled. When the index of the vernier is in the horizon, or nadir, as the case may be, the control bubble is centered with its own adjusting screws. Conversely, when the control bubble is centered, the verniers of the vertical circle will be adjusted to a position to indicate correct vertical angles, irrespective of slight displacements in the rest of the instrument.

Whenever an important vertical angle is to be observed, as in stadia topography, the control bubble should be consulted and centered with the Index Adjuster. In some instruments this is accomplished with two capstan screws, but for ready service our construction is such that the bubble can be centered at any time with the small tangent screw operating against a post in the standard. The little check nut is to be first loosened and need not be used at all thereafter unless the operator chooses to do so. The check nut is used in the original adjustment to clamp the cover guard and vernier scales in a position normal to the rest of the instrument and to prevent insidious errors by accidental interference with the index adjuster.

The adjustment of the control bubble is contingent upon the seven distinct processes touched upon between pp. 62 and 72. When the horizontal cross wire is properly collimated and the telescope bubble adjusted to it, the index lines of the vernier scales of the vertical limb must be made to coincide with those of the circle.

This is accomplished with the little tangent screw called the "index adjuster". It is provided with a check nut so that accidental movement will be prevented. This accomplished, the control bubble is adjusted to these conditions by simply bringing it to the center of its scale with its own adjusting screws.

As the lower portion of the instrument departs from ideal conditions, for any of the well known reasons, the control bubble will naturally deviate from its normal position. In such a case it is to be re-centered just before any sight involving a vertical angle, by the method previously described.

The index adjuster is to be utilized continuously for this purpose without reference to conditions which prevail in the rest of the instrument.

Graduations and The Vernier.

The limb, or principal scale of the portable transit instrument, may not be conveniently divided into spaces that are smaller than $1/2°$ to $1/3°$, and were it not for the supplementary scale, called the "vernier", readings could not be taken any closer than is possible with the ordinary compass. The device used to read the fractional parts of the subdivision of the limb is named after its inventor, Pierre Vernier, who introduced it in 1631; but it did not become practical until Jesse Ramsden invented the automatic dividing engine in 1768. In some foreign countries the scale is still known as the "nonius", after Pedro Nunez whose method of reading the quadrant (1542), however, was totally unlike the scale here considered.

The vernier may be applied to either curved or straight lines. It consists of a small movable auxiliary scale whose object is to provide a ready means of estimating the fractional parts of the main

scale without going to the extremity of subdividing the limb into microscopic spaces.

The principle of the vernier depends upon the simple proposal to take n number of divisions on the limb and of dividing a space of equal length into $n + 1$ equal parts. Practical considerations as to the space available and the legibility of reading, as well as the least count desired, are the influences which determine the value of n.

The least count is the quotient of the value of the smallest division on the limb (l), divided by the total number of divisions on the vernier ($n + 1$), representnd by the formula: $x = l \div (n + 1)$.

In the inserted example, the space equal to nine subdivisions on the scale is redivided into ten equal parts. One space on the vernier is therefore equal to $\frac{9}{10}$ of a space on the limb, or one-tenth shorter; hence the distance ab is $\frac{1}{10}$ of the value of the smallest division on the limb, and cd would be $\frac{2}{10}$, etc. If the vernier scale is raised until a coincides with b, the index line will be $\frac{1}{10}$ of a division above the figure 3, and the reading will be 3.01. In the second drawing, in the same way, we have 3.02, but in the third drawing the index has been moved above the second tenth-mark on the scale and the sixth line of the vernier is in coincidence. The reading is therefore 3.26.

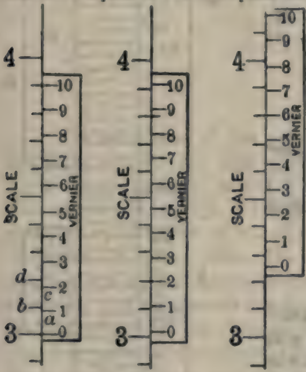

Fig. 53
(From Johnson-Smith)

The value of the least count in this example will be determined by dividing the value of one subdivision on the scale, $\frac{1}{10}$, by the number of subdivisions on the vernier, 10, or $\frac{1}{100}$. The zero of the vernier is the index line which indicates the position on the main scale of which the linear or angular value is desired. The reading begins by first counting the number of whole divisions on the main scale then running the eye along the vernier scale until a coincidence is found.

The width of line is not necessarily dependant upon the width of marginal space occupied by the least count on the periphery. Accuracy depends more upon the ability to judge when two lines are in coincidence than the attempt to deal with actual relative values. If the width of the graduation line and the spaces between them are absolute and uniform, then legibility is more important than any other consideration.

A Direct Vernier is one in which the numbering of the scale increases in the same general direction with the numbering of the limb. The direct verniers, as used with transits, are

either single or double, depending upon whether the limb is supplied with one or two rows of figures. If angles were always read from left to right, there would exist the necessity for only one row of figures and a single vernier by which the possibility of error in reading would be nearly, if not quite, overcome.

Examples of direct verniers, each having a least count of 0.01, are given at B, D, and F in Fig. 54 which is reproduced from an article by G. H. Bainbridge Jr. in *Eng. News*, Jan. 25, 1912. Other examples of direct verniers as applied to transits are shown in Figs. 55 and 59.

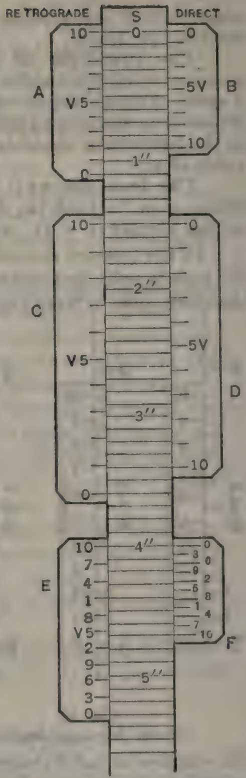

Fig. 54

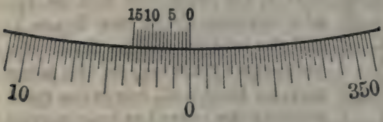

Fig. 55

Fig. 55 shows a single vernier, reading to minutes, applied to a circle with one row of figures reading clockwise consecutively from 0° to 360°. This method of numbering is most popular for mining theodolites. With extra heavy and legible lines the difficulties ordinarily attendant upon that class of work are reduced to a minimum.

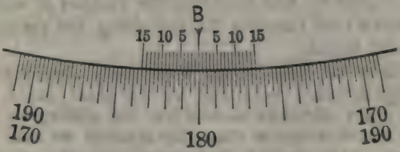

Fig. 56

Fig. 56 shows a double vernier of the same construction with a scale running each way from a central zero point or index line There being two complete verniers in this scale, there are always two lines in coincidence, which adds something to confusion and the consequent possibility to error.

L. E. Picolet in *Eng. News*, Apr. 1912, proposed a method of separating double verniers, as shown in Fig. 57. In this system the index line was made very prominent with a clear space of slightly more than one degree at each side so that, while it performs the function of the zero line common to both scales, the scales themselves are separated, giving emphasis to the direction in which the angle is to be read.

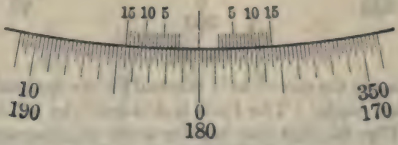

Fig. 57

The zero-lines of the separate scales, if they existed, would occur at exactly one degree, or any desired number of integral divisions to the right or left of the index line. The least count in this vernier, as in Figs. 55 and 56, may be found by dividing the smallest subdivision of the limb, 15′, by the whole number of divisions of either scale, 15, or 1′. As a rule, however, minute graduations are not accomplished by any such fine subdivision of the limb. ½° spaces on the limb and a vernier of 30 spaces equal to 29 on the limb, is the most legible form for minute graduations. ‡ Another method of obtaining the smallest count of 1′ is to divide the limb into ⅓° spaces and construct a vernier scale by dividing 19 such spaces into 20 equal parts.

In 1801 Laplace suggested that the quadrant of the circle should be divided into 100 degree-spaces (= 0° 54′ Sex.) and that each of these centesimal degrees, or *grades*, should be subdivided into 10 minutes, or 100 seconds; but the sexagesimal system already universally adopted has not yet been widely replaced even though the tremendous work of making the translation and compiling logarithmic tables has been accomplished. *

Henry Briggs was the first to use the sexagesimal degree, decimally. His *Trig. Brit.*, edited by Gellibrand, was published in 1633. This method has the great advantage of dispensing with both minutes and seconds without destroying the system itself. In 1856 M. Minot popularized it in France for laying off deflection angles in railway curves, and S. W. Mifflin used it with success in the early construction of the Pa. R. R.

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‡ *Refer also to Mines and Minerals, June 1902, p. 525.*

* *Tables Trigonometrique Centesimals, J. L. Sanguet, Paris, 1889.*

Prof. E. V. Huntington, of Harvard, has published a book of four place functional tables in which the sexagesimal degree is divided decimally. * There is the same argument for using the decimal degree as there is for the decimal foot.

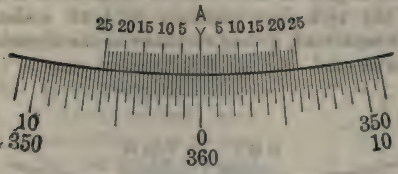

Fig. 58

Fig. 58 shows the most likely method for the decimal subdivision of a sexigesimal circle. The limb is divided into ¼° or .25° spaces exactly as in Fig. 56; but the vernier scale is constructed by subdividing 24 such spaces into 25 equal parts so that the least count will be 1/25 of ¼° or 1/100°. In reading this type of vernier it is necessary to retain in the mind 25-, 50- or 75-hundredths of a degree before seeking the *addendum* in the vernier scale. By this method Ver. A. could be constructed as above and Ver. B. as in Fig. 56 so that both the decimals of a degree, and minutes, could be read on the same circle.**

A Retrograde Vernier is one in which the numbering of the scale increases in the opposite direction or one in which the scales spaces are larger than those of the limb. Examples of retrograde verniers are given at A, C, and E, Fig. 54 showing three other methods of producing a least count of 0.01. The verniers at E and F are unusual.

A Reciprocating or Folding Vernier partakes of the nature of the direct and retrograde types, in that it may be read both forward and backward. Two examples from our catalog Metro III are reproduced herewith. The system and values are exactly alike in each case but the smaller illustration, Fig. 59, shows the method of numbering when only one zero point is permissible, as in reading the quadrants of a vertical circle.

Let it be supposed that a reading is to be taken downward or toward the left. Start with the horizon at zero and follow along the scale in the direction of 5 and 10. If no line is found in coincidence, jump at once to 10 at the opposite end of the scale and continue the count in the direction of 15 and 20. Where space is cramped the same effect can be produced on small vertical circles, reading to minutes, with a limb divided into ½° and an ordinary single vernier of 30 spaces equal to 29 on the limb.

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* *Harvard Co-op. Soc., Cambridge, Mass. 1907, '08, '10.*

***See also Railroad Field Manual, W. G. Raymond, 1915, p. 122.*

Fig. 59

Colored Figures

In the larger illustration the method of reading the scale is more pronouced. We observe the lower row of figures in reading toward the left, and the upper row in reading toward the right. As in Figs. 56, 57 or 58 and in all cases where double verniers are used, there is offered to those who are less experienced, the constant invitation to confusion and error in reading the wrong vernier. The first precaution devised to overcome this hazard was to incline the figures of both the limb and the corresponding vernier in the direction of observation. In 1910 we introduced the additional precaution of coloring the R. to L. limb and its corresponding vernier figures in carmine as indicated above.

Waterproof Construction

Waterproof construction in every detail has been a recognized necessity for a great many years. The vernier windows are set in cement, the glass cover of the compass box is "easily removable and yet water tight", † the clearance between the horizontal plate is reduced to less than 0.1 mm which excludes both dust and moisture and the interior system of focusing has made it possible to protect

† *W. L. Cumings in Eng. and Mng. Jour. June 20. 1914, p. 1240.*

the telescope the same way and so prevent the spider webs from sagging or wrinkling in damp weather. If, in the course of time the telescope lenses seem to loose their lustre, unscrew the objective, rack out the focusing mechanism, remove focusing pinion if necessary and unscrew the inner lens barrel. Dust particles should be removed with a camels-hair brush, but dirt is best removed by a soft rag moistened with gasolene and dried with soft linen.

Whenever a lens system is disturbed, however, both the collimation adjustment and the stadia interval will most likely be affected. Take, for instance, an objective in one of our tachymeters in combination with the focusing lens which has an equivalent focal length of say 176 mm. The space between the stadia wires must be 1.76 mm. When it is understood that these intervals must be figured to the nearest 1/100 mm, one can appreciate the delicacy required in mounting.

Alignment and Eccentricity

The circle is provided with opposite verniers to correct for small errors in the spacing thereof, but the index line of each vernier should properly lie in a straight line through the center of the azimuth axis. This setting is accomplished by the maker with the aid of two diametrically opposite microscopes and is generally conceded to be both accurate and permanent. The mechanical test of this setting depends upon the accuracy of spacing between the graduation lines and shows only a probable error, whatever the size. The test for eccentricity of setting is developed in *Merriman-Brooks Handbook,* begining at p. 82.

Obviously there would be two opposite positions, that will never show eccentricity, when the line passing through the zeros of the verniers also passes through the center of the azimuth axis. We keep the errors of eccentricity reduced to a few seconds of arc. This is a matter that need not seriously concern the average surveyor because errors of this size will not be distinguishable or effective in circles graduated to read minutes of arc. It is recorded in *Johnson-Smith* at p. 81 that the error of eccentricity involves no appreciable error in measuring horizontal angles. An eccentricity of 1/1000 of an inch in the azimuth axis will cause a maximum error of 1′ 08″ on a 6-inch circle if but one vernier were read; but a mean of both verniers eliminates all errors due to this cause.

Fortunately for the better class of field work, as well as for precise triangulation, much of the effect of this recognized source of error can be eliminated by the well known process of repetition — a certain number of times with the telescope erect and an equal number of additional observations with the telescope inverted.

The accuracy of spacing, centering and alignment having been established for any particular make of transit, most engineers will

accept conditions founded upon the reputation of the graduating engine used by the manufacturer, and for ordinary work will read only one vernier.

If two verniers are not set exactly 180° apart the error of alignment can be overcome first, by using only one vernier, which keeps the error constant and negligible and second, by taking the mean of two vernier readings at every observation.

The Stadia

The invention of this tachymetric principle should be properly attributed to G. Montanari who published at Cologne in 1674 and at Venice in 1680, a method of placing many equidistant filaments of known value on the diaphragm *.

The Danish Acad. of Sci. awarded a prize to G. F. Brander for the stadia he used with an alidade in 1772, and in 1777 the Soc. of Arts presented an award to William Green of London for the same invention. The great inventor and engineer, James Watts, however, was known to have used the stadia interval as we use it today for field work in 1770 **. Prof. Fontana of Florence proposed spider lines instead of human hair or silk filaments in 1775. Ramsden was doubless the first to mount spider webs and Rittenhouse of Phila. followed his example in 1786. The first extensive use of the stadia method in the U. S. was organized by J. R. Mayer in connection with the Great Lakes Survey in 1848.

When the transit instrument is supplied with equally and accurately spaced stadia *** wires, a telescope bubble, and a vertical limb, it is capable of the rapid location of points by the polar coördinates of azimuth, elevation and distance, with reference to some datum, which gives it the right to be designated as a "Tachymeter". If the telescope possessess sufficient illumination and power the stadia method is even more accurate than the chain, and if the telescope bubble is correctly associated with a properly constructed sight-line, the percentage of error in elevation will be even less. Chaining errors are cumulative while the errors of observation in stadia surveying tend to compensate one another ****. On this account the stadia has very rapidly superceded all other topographic methods for small scale maps.

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* *Geometria Applicata, A Salmoraghi, Milan, 1884, p. 278.*

** *Prof. J. L. Van Ornum, Bul. of Univ. Wis. Vol. 1 p. 354.*

*** *The word* **stadia**, *being the plural of* **stadium**, *is of Roman or Greek extraction. It was 1/8 of a Roman mile. Porro used the word "stadia" in 1820 to designate the rod, as originally intended, but it has now come to signify the method only.*

**** *Prin. and Pract. of Surv., Breed & Hosmer, 1908, Vol II, p. 147.*

The instrument should have a vertical limb of superior construction and its index error should be either allowed for, or regulated constantly with a **Control Bubble.** For stadia work the control bubble (see p. 94) is nearly indispensible.

The Accuracy of Stadia Surveys depends upon the fineness of the wires, the power, illumination and character of the field in the telescope, the length of sight,* atmospheric conditions, the value and stability of the wire interval, the nearness of the sight to the surface of the earth, the personal equation, the kind of graduations on the rod and the number of observations or the extent of the survey.

The size of the web runs from .0001 to .0003 in. The latter is considered coarse if under high magnification, but some prefer them so, to distinguish them from the finer central cross wires (see also p. 144).

Theory

The principle upon which the stadiametric distance is determined is the simple geometric proposition that the homologous sides of similar triangles are proportional. In Fig. 60 let the two dots at the diaphragm, F, represent the stadia wires. The focal length of the objective being F, let the interval between the wires be one one-hundredth of that amount and let it be represented by

$$F \div 100 = ab = i.$$

Every lens has two focal planes. In this case let the anterior focal plane be designated by F_1 so that $OF_1 = FO$. In any telescope the lines of vision are not projected from the eye but rather there is received an impression of the field of view. Then let it be assumed that a rod being placed a certain distance from the instrument, a ray of light proceeding from the point A, will pass through F_1, on to a, then by refraction through the objective, finally pass the upper stadia wire and out through the eyepiece somewhat as indicated. The same course of reasoning is true for the point B and it is obvious that the further the rod from the instrument the greater will be the interval and that this interval, I, will always be proportional to the distance of the rod from F_1. Then,

$$ab : OF_1 :: I : D \quad \text{or}$$
$$i : f :: I : D$$

by which it appears that the origin of the distance, D, is at F_1 and not at the center of the instrument, and further that

$$D = I\,(f \div i)$$

$f \div i$ is a constant factor for each instrument, usually known as K, corresponding to 100 more or less closely. Our method of

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* *L. H. Goodwin in E. & M. Jour., Nov. 24, 1914, p. 957 sets 700 ft. as the limit by which the probable error < 1:800.*

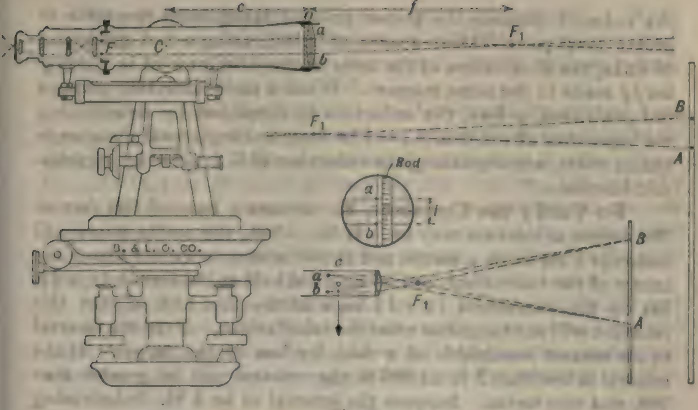

Fig. 60

spacing the stadia wires gives uniformly reliable results. They are equally spaced above and below the central wire so that distances between a ¼ and ½ mile can be estimated with the ordinary leveling rod, by the half interval of 1:200.

The constant (c + f), or the distance from the center of the instrument to the anterior focal point, was a slightly variable factor due to the process of focusing with the older style of draw-tube, until it was made uniform by our cryptic focus. It is applied to all observations of whatever size, irrespective of whether the whole or half wire interval is used. It may be determined, by focusing the telescope on some distant point, then by measuring the distances from the center of the objective back to the diaphragm, adding to this the distance from the objective back to the center of the telescope axis.

The true formula for calculating all horizontal distances by the stadia method is :- $D = KI + (c + f)$ but the last factor is sometimes ignored on long sights if the allowable error will permit it. For certain classes of work we occasionally fix the interval, by request, so that it will read exactly 5 : 500. If it reads 5 ft. on a rod held 500 ft. from the center of the instrument, the interval is obviously larger that it should be. All distances read by such an interval, except the one for which the computation was made, will be slightly in error.

The specifications of the General Land Office call for a fixed stadia interval of 1 : 132. We quote from the text in the Department *Field Manual*:

"In public land surveying it is convenient to have fixed stadia wires with a ratio of 1:132 so that the sum of two rod readings in feet will be equivalent to a ratio of 1 : 66, or a reduced distance in chains; it is also convenient to reduce the error in the wire interval to the error in 10 chs., and to eliminate the error by applying to

the reduced distance the proper amount taken from the table of proportional parts. With a ratio of 1:100, using a rod graduated to links, the elimination of the error in the wire interval is conveniently made in the same manner. With a ratio of 1:100, using a rod graduated to feet, the reduction is simplified by determining the logarithm of the true K, rod in feet, and horizontal distance in chains units, accomplishing the reduction of $K\,I\,\cos^2 v$ by logarithmic functions."

To Verify the Constant, K, measure a base of 500 feet or 200 meters as the case may be. If $K = 100$, the stadia wires will intercept 1 meter on a metric rod at 100 meters $+ (c + f)$ from the center of the instrument, or 1 ft. on a rod held at 100 feet $+ (c + f)$. Set up the instrument $(c + f)$ back of the first hub so that this constant will not thereafter enter the calculations. Read the interval to the nearest hundredth of a foot by use of the target. If the interval is less than 5 ft. at 500 ft. the constant will be greater than 100, and *vice versa*. Suppose the interval to be 4.95. Substituting in the value of D (p. 104) we have:

$$500 = 4.95\,\frac{f}{i}, \text{ or } \frac{f}{i} = 500 \div 4.95 = 101 \text{ or } 1:101$$

Intermediate points may also be checked up and an average struck, but the atmospheric conditions should be as nearly as possible alike. Prof. L. S. Smith has shown * that differential refraction between the upper and lower wires will make the intercept on the rod greater in the morning and evening than at midday by a variable error up to .004 ft. per ft. of interval, and that this phenomenon is more noticeable near to the ground than in the air strata some distance above. It is a most fruitful source of error in stadia surveying. He therefore lays down the following rules:—

1. *Every instrument man should determine for himself his wire interval or make observations for graduating special rods.*
2. *Determine the wire interval for various distances but only between the limits expected in field work, and for several hours distributed through one or more days, under conditions which do not differ radically from the country to be surveyed.*
3. *For a radical change of field or season conditions, re-determine the wire interval or rod graduation.*
4. *Avoid reading the lower cross wire near the ground either in the interval determinations or in the field work, but the interval determination readings should agree in this respect with the average field practice.* We add:
5. *For reading distances under 1300 ft., set the lower wire on an intergral foot division; count the number of feet, tenths and hundredths to the upper wire and estimate the fraction. Readings taken with whole intervals are twice as accurate as with half intervals, but when the sum of the half intervals equal the reading with the whole interval the check is significant.* **

* *Bul. Univ. Wis. Vol. I No. 5, 1895.*

** *J. A. McDonald in Canadian Engineer, July 30, 1914.*

The diaphragm mount should be very substantial. Weak mounts have caused serious discrepancies. E. McCollough reports * a "K" variation in the same instrument, due to various causes, ranging between 101.5 and 102.7.

The most permanent interval is to be secured by etching lines on a glass diaphragm. In the erecting telescope of the older style with Fraunhofer or Dollond eyepiece this expedient is rather objectionable because of the inaccessibility for cleaning; but with the inverting telescope the ocular can be easily removed for this purpose and with the Huyghens ocular, which we use in conjunction with the achromatic triplet erecting system, we preserve the erect image of the field as well as accessibility to the diaphragm.

The fact that the constant, K, is not always exactly 100 and the fact that damp weather is likely to sag the spider lines and affect the ratio, gave rise to the demand for Adjustable Stadia now met in another way by our cryptic focus and dry telescope interior.

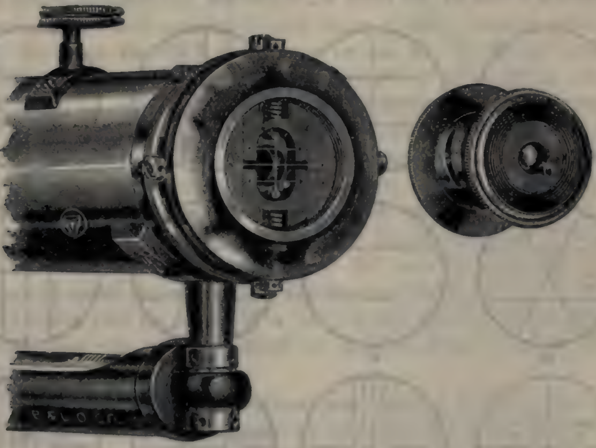

Fig. 61

Adjustable Stadia Wires are arranged so that the operator may test and adjust his wire interval with the same regularity that other parts of the instrument are rectified; but experience has repeatedly confirmed the opinion that the more readily some adjustments are to be made, the more necessary it becomes to make them. The late Prof. J. B. Johnson, used to say that if fixed stadia sometimes change, no argument should be advanced for avoiding the adjustable sort. This fact may also be logically drawn, nevertheless,

* *Eng. Rec. May 11, 1912.*

for their adoption. The argument rests mainly on a matter of personal preference, but the fixed stadia have been heretofore overwhelmingly preferred.

It is generally understood that the fixed stadia are not intended for *precise* work, and if it is desired that the error shall be reduced below 1:800 * it is a generally accepted requirement, quite apart from the original accuracy of spacing, that a special rod should be graduated for each instrument ** or the interval factor should be occasionally checked up and revised. There is much to be said in favor of the adjustable stadia and, while condemned by several authorities, they might be more widely and successfully employed. Recently we made for export a diaphragm with a fixed interval of 1:200 below the central cross wire and an adjustable interval above, ranging between 1:200 and 1:1000.

To Regulate the Adjustable Stadia, level the telescope, as nearly as may be, and place a rod exactly at 100 ft. plus $(c + f)$

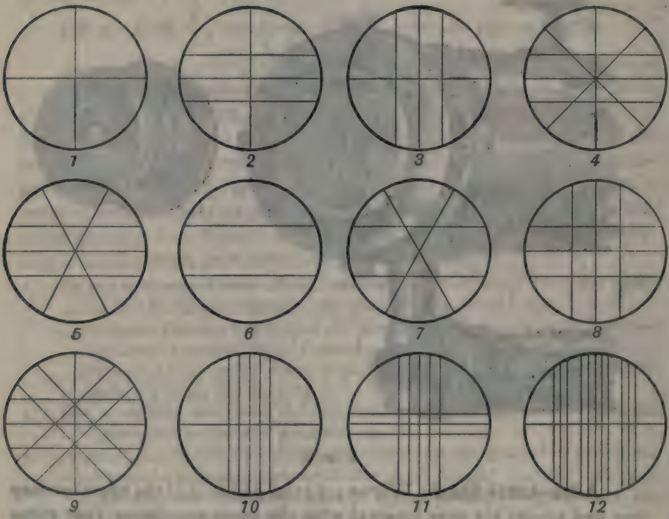

Fig. 6a

* *The U. S. Engrs. on the Survey of the Gt. Lakes have secured an accuracy of 1 : 1888 and the Idaho Boundry Survey reached 1 : 2560 while others have attained even better results; but W. Newbrough of Evanston, Wyo. reports that for filling in details in mountainous districts a greater accuracy than 1 : 400 is not specially attempted. Relative accuracy tends to increase with the length of sight.*

** *In Eng. Rec. May 25, 1912, J. W. Woermann, U. S. Ass't. Eng. reports the use of 13 transits of different makes, each with a specially graduated rod.*

from the center of the instrument. Note where the central collimated horizontal cross wire intersects the rod. Set two targets, if convenient, exactly 0.5 ft. above and below this middle position. Turn the stadia adjusting screws, shown in Fig. 61, until the wires bisect the targets so placed. Repeat the test at 200 ft. plus (c + f), at 300 ft. plus (c + f), etc. If the metric system is used, the test targets should be spaced 0.5 m above and below the middle position at 100 m plus (c + f), let (c + f) be expressed as it may.

Disappearing Stadia were devised in 1880 by Verplanck Colvin, then N. Y. State Surveyor. When the stadia wires are mounted on the same diaphragm with the regular cross wires, as in cut 2, Fig. 62, there is some chance of using the wrong horizontal wire for leveling or for observing vertical angles, particularly in the dusk or under similar unfavorable circumstances. We mount the ordinary disappearing stadia on the same reticle with the regular cross wires but in a different focal plane, so that when the eyepiece is refocused the field appears as in cut 6. It is not strictly necessary, but most engineers prefer some sort of centering device as in cut 7. These wires are commonly known as the St. Andrew cross, and while they are accurately centered to the same alignment with the principal cross wires, they do not, however, provide for so great assurance in reading half-intervals, or vertical angles, so that the arrangement shown in cut 5 is frequently specified. Occasionally also a single vertical wire is added to the combination shown in cut 7 to assist in plumbing the rod. For mining work the disappearing stadia are commonly used to avoid confusion in the dark, but where they are preferred in the principal focal plane the central horizontal wire is distinguished by a set of diagonal wires, as in cut 4.

Inclined Stadia Observations require a double correction. The fact that the sight is inclined demands that the observed distance shall be reduced to the horizon. If the rod is held vertically, as it usually is, then it is not normal to the sight-line and too large an interval will be read, which must also be reduced. The objection to holding the rod inclined and normal to the sight-line is the judgment which must be thus delegated to the rodman, as well as the complications which would arise in getting elevation by this means.

The interval, AB, observed, is greater than the true reading, *rs*. It will appear from Fig. 63 that AMr = BMs = v. For most observations the interior angles at *s* and *r* approach 90° closely enough so that BsM and ArM may be treated as R.A. triangles.

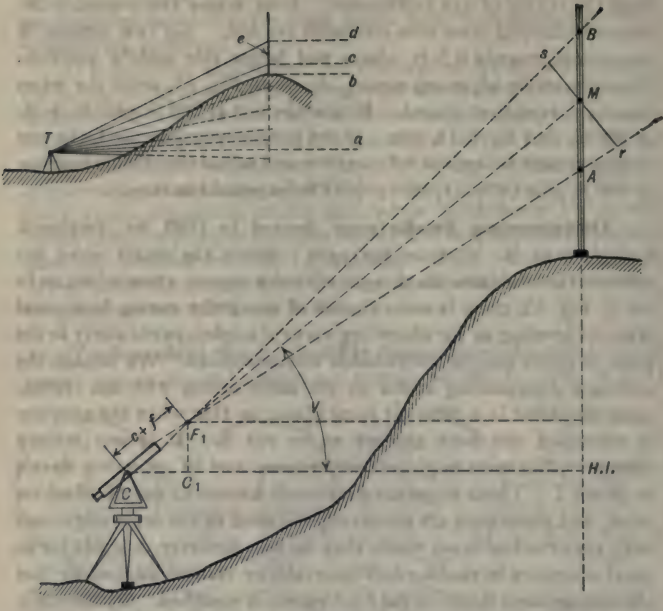

Fig. 63

Hence AB $\cos v = sr$. In other words, the interval read must be multiplied by the $\cos v$ to get the rod reading from which F_1M may be deduced. Then,

$$CM = F_1M + (c + f) = (c + f) + K\,(AB \cos v)$$

$$CC_1 = (c + f) \cos v \text{ and } C_1\text{-H. I.} = F_1 M \cos v; \text{ but}$$

$$F_1M = K\,(AB \cos v), \text{ therefore}$$

$$\text{C-H. I.} = (c + f) \cos v + K\,(AB \cos v) \cos v$$

$$= (c + f) \cos v + K\,(AB \cos^2 v)$$

Let I = actual reading, or interval AB; then,

$$H = \text{C - H. I.} = K\,I \cos^2 v + (c + f) \cos v.$$

For the vertical component the distance H. I.-M is required. H. I.-M = CM $\sin$ v, and substituting the value of CM as above,

$$\text{H. I.-M} = [K\,(AB \cos v) + (c + f)] \sin v$$

$$= K\,(AB.\ \cos v.\ \sin v) + (c + f) \sin v, \text{ but}$$

$$\cos v . \sin v = \tfrac{1}{2} \sin 2v, \therefore$$

$$V = \text{H.I.-M} = K\ I\ \tfrac{1}{2} \sin 2v + (c + f) \sin v$$

In using the formula for **H** and **V** it should be born in mind that $(c + f) \cos v$ will not differ materially from $(c + f)$ up to 5° elevation, and that $(c + f) \sin v$ is negligible for ordinary topography up to this limit. With our cryptic focusing telescopes $(c + f)$ is so reduced that this limit of negligence can be extended to 9° or 10°.

For reading inclined observations, the center mark, M, should be as high above the peg as the instrument is above its station but this is frequently averaged at 5 or 5½ ft. depending upon the stature of the observer. For greater accuracy the H. I. Plummet, illustrated and described on page 176, will be found convenient.

One method of overcoming complicated formulæ for **H** is to place vertical stadia wires in the telescope, as in cut 3, Fig. 62, and to have the assistant hold the rod *steadily* in a horizontal position, This will give the exact inclined distance which can be reduced directly to the horizon by the cosine of the vertical angle. The vertical stadia are occasionally specified for levels to be used in this way. E. M. Douglas, U. S. G. S., proposed sometime ago* that the stadia wires should be mounted on a revolvable diaphragm for this purpose, but the idea has not been generally adopted for fear the collimation adjustment might be disturbed in the process. The better and cheaper plan, tor levels would be to adopt the diaphragm, as in cut 5, Fig. 62.

Stadia Reduction Tables contained in the Appendix of this publication are widely used in preference to diagrams or slide rules. That part of the compilation up to and including 30° of altitude was computed by Arthur Winslow for the 2nd. Geol. Surv. of Pa., 1874-87, and that from 30°, up, by H. N. Evanson of Pittsburgh, some 20 years ago. We add the $(c + f)$ corrections for .40 and .60 ft. in each case to facilitate work with the lower constants of our alidades and our 4½ and 5-in. Tachymeters.

A. P. Davis, Ch. Eng. U. S. Irr. Dept., and C. G. Anderson of the U. S. G. S., have also computed tables for special work in their respective departments, so that desired results can be obtained by addition instead of multiplication.

The Reduction Tables on page XXVII in the Appendix are taken from the *Army Manual.* They are intended also for the reduction of the inclined measured distance to vertical and horizontal components, but in order that the reduction shall be free from error it is recommended that the rod must be inclined until normal to the line of sight. For this purpose erect a perpendicular pointer on the target and slide it along on the rod to the height of the eye. The rodman should aim the pointer at the instrument when the sight is being taken. If the rod is held vertically, these tables will produce errors of 1% at 8°, 2% at 11°, 3% at 14° etc.

* *Eng. News, May 17, 1906.*

If the rod is held inclined, however, there will be a slight error in distance due to the fact that the central wire does not intersect the rod at a point directly over the station.

The "Interval" or "Stepping" Method of Determining Elevation

The only references we have noticed setting forth this method were published in the *Eng. News*, N. Y., April 28, 1910 by E. M. Douglas; Sept. 1, the same year by A. F. Meyer, and in *Eng. and Cont.*, Chicago, Feb. 18, 1914, by H. H. Edgerton.

By reference to Mr. Meyer's cut, which is the upper portion of Fig. 63, the several stadia intervals are to be laid off from the horizon against "stepping points" in any visible back-ground, above or below the instrument, until the rod appears in the field of view.

Let:—

ab = difference in Elev. between H. I. and Sta. on hill;

c = reading of upper wire at n intervals

cd = rod interval = I

$cd \cos v$ = true interval

$Tc = 100\ (cd \cos v)$, nearly;

$cTa = n\ (34' \, 23'')$ if K = 100

$ab = (Tc \sin cTa) - bc$;

$= n\ (cd) - bc$, approximately.

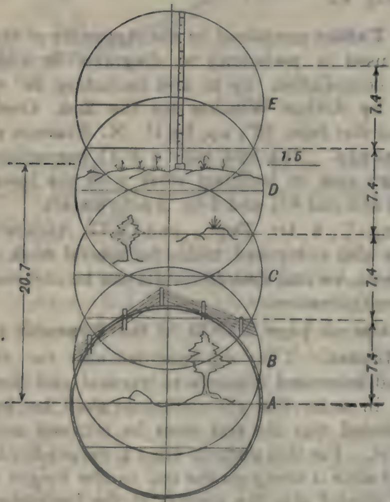

Fig. 64

n (cd) will be sufficiently accurate for most purposes up to n = 3 Beyond this apply Meyer's Rule as follows:-

When more than three intervals are read, reduce n (cd) or nI by 1% of itself for each interval above three.

Fig. 64, being an adaptation from Mr. Edgerton's article, shows the original field, A, which is in the horizon. In field B the telescope has been raised so that the lower stadia wire cuts the original horizon point; then the telescope is raised successively, by steps equal to the stadia interval, until the upper stadia wire cuts the rod, as in field D. The stadia interval is finally found, approximately, as in field E, setting one wire at any convenient foot mark. We then have a total intercept of n I ± some overlap.

Stadia Rods have been designed in extensive variety. While no one pattern may be best suited to all classes of work, the saw-tooth or diamond point designs have been widely adopted so that fractional readings can be made by estimating the position of the wire against a diagonal line. Simplicity rather than multiplicity is the chief requisite. Ideal conditions provide for an unchangeable wire interval of exactly 1:100 and a rod that will answer for stadia work as well as for leveling.

Beaman Stadia Arc

This device was designed by W. M. Beaman, U. S. G. S., for the mechanical indication of those angles which are simple multiples of the differences in elevation, expressed in rod intervals. Mr. Saegmuller made the first model for Mr. Beaman in Washington in June, 1904, to carry out the rapid instrumental execution of the "Stepping Method".

The first stadia interval above the horizon, as in field B, Fig. 64, has an angular value of 1:100, or 34′ 23″. As the gradient becomes more precipitous and the number of intervals increase, the angular value of the interval increases as the cord of the stadia angle, or ½ *sin* $2v$, as previously deduced, so that when the tenth interval is reached, for instance, the value of that interval is 35′ 01″ and the total angle is 5° 46′ 07″ instead of 10 × 34′ 23″.

In this manner we may graduate a supplementary arc, known as the V-scale, in which the spaces gradually increase both sides of a center line, arbitrarily numbered 50, so that the notes will show, beyond doubt, whether the vertical angle was one of elevation or depression. V—42, for instance, would be an 8-interval angle of depression while V—58 would be an 8-interval angle of elevation.

To Utilize the V-Scale for determining elevations, turn the telescope up or down until a full stadia interval can be measured on the rod when the index coincides with some division on the V-scale. Note the interval, also the position of the central wire as well as the division indicated on the V-scale.

Let it be assumed that the interval was 8.45, that the central wire cut 6.30 above the peg and that the V-scale indicated 57. This being an angle of elevation, the computation would be:—

$$57 - 50 = 7 \times 8.45 = 59.15 - 6.30 = 52.85$$

which is the elevation of the peg above the H. I.

To test the accuracy of the V-scale, we append a table of equivalent values. Set the principal arc, if an alidade, at 30°. Set the Beaman Arc at 50. As we make them, this will place the scales in proper relative position.

By use of the telescope clamp, turn the Beaman Arc up 1, 2, 3, 4 divisions, etc., in succession as in the first column and compare the angle indicated on the principal arc, which should correspond with the values given in the second column. These are computed from the well known formula: ½ *sin* 2v.

No. of Interval	Angle (½ Sin 2v)	Dif. in Minutes	No. of Interval	Angle (½ Sin 2v)	Dif. in Minutes	No. of Interval	Angle (½ Sin 2v)	Dif. in Minutes
	° ′			° ′			° ′	
0	00.00							
		34.38			36.16			43.38
1	0 34.38		16	9 19.89		31	19 09.48	
		34.39			36.42			44.27
2	1 08.77		17	9 56.31		32	19 53.75	
		34.42			36.70			45.25
3	1 43.19		18	10 33.01		33	20 39.00	
		34.47			37.00			46.31
4	2 17.66		19	11 10.01		34	21 25.31	
		34.52			37.32			47.50
5	2 52.18		20	11 47.33		35	22 12.81	
		34.59			37.71			48.82
6	3 26.76		21	12 25.04		36	23 01.63	
		34.68			38.08			50.31
7	4 01.44		22	13 03.12		37	23 51.94	
		34.77			38.49			51.99
8	4 36.21		23	13 41.61		38	24 43.93	
		34.88			38.95			53.89
9	5 11.09		24	14 20.56		39	25 37.82	
		35.02			39.44			56.08
10	5 46.11		25	15 00.00		40	26 33.90	
		35.16			39.97			58.64
11	6 21.27		26	15 39.97		41	27 32.54	
		35.33			40.54			61.66
12	6 56.60		27	16 20.51		42	28 34.20	
		35.50			41.16			65.30
13	7 32.10		28	17 01.67		43	29 39.50	
		35.71			41.85			69.77
14	8 07.81		29	17 43.52		44	30 49.27	
		35.92			42.58			75.47
15	8 43.73		30	18 26.10		45	32 04.74	

In work, it occasionally happens that no multiple setting can be found that will throw the middle wire on the rod. In this case, take a half interval, at any convenient position. Let this be $7.5 \times 2 = 15$, or 1500 ft. Now turn the telescope to the nearest indicator and let it be assumed that the lower wire cut the rod at 8.4. The computed middle wire reading will then be $8.4 + 7.5 = 15.9$. Proceed as above.

Note—In none of these calculations is $(c + f)$ taken into consideration. All computations begin at the anterior focal point and

the instrumental constant for any angle, as determined by the stadia tables, is to be finally added if the accuracy attempted requires that the constant be considered at all.

The Beaman Stadia Arc

as applied to the Standard Alidade

Fig. 65

The H-Scale is used for the rapid reduction of horizontal distances, from the observed inclined distance, as determined from the rod interval and the instrumental constant. This is a second set of graduations representing certain angular values that correspond to those in the reduction table on page 116. Obviously there will be definite angles of inclination in which the base can be determined directly by substracting from a hypothetical hypothenuse, a certain percentage of itself; thus, if the telescope is inclined at 18° 26′ and the uncorrected stadia interval read 7.2 (or 720 ft. inclined distance) we know that the horizontal equivalent is to be determined at once by substracting from this imaginary distance, 10% of itself or 72 ft., leaving 648 ft. as the correct horizontal distance to the anterior focal point.

The V-Scale and the H-Scale do not correspond precisely but when the V-Scale has been set to some even multiple, as explained, the graduations of the H-Scale will suggest the approximate if not the actual percentage by which the inclined distance is to be reduced. If necessary, slightly change the inclination of the telescope to correspond with one of the angular values given in the tables, then substract the corresponding percentage from K times the rod interval.

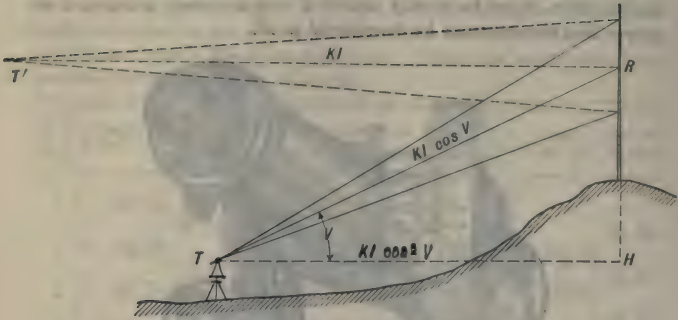

Fig. 66

Referring to Fig. 66, the problem definitely stated is:— For what vertical angle (v) is the line KI $\cos^2$ v, n% of KI shorter than KI? Now,

$$n\% = n \div 100 \text{ and}$$
$$KI \cos^2 v = KI - (n\%\ KI)$$
$$\cos^2 v = 1 - (n \div 100)$$
$$\cos v = \sqrt{1 - (n \div 100)}$$

cos **v** will therefore equal the square roots of .99, .98, .97, etc. The table following is computed from this formula.

%	°	′	″	%	°	′	″	%	°	′	″
½	4	03	17	8	16	25	48	19	25	50	31
1	5	44	21	9	17	27	27	20	26	33	54
1½	7	02	06	10	18	26	06	21	27	16	29
2	8	07	48	11	19	22	11	22	27	58	20
2½	9	05	51	12	20	16	04	23	28	39	29
3	9	58	27	13	21	08	03	24	29	20	02
3½	10	46	58	14	21	58	22	25	30	00	00
4	11	32	13	15	22	47	11	26	30	39	26
5	12	55	15	16	23	34	42	27	31	18	23
6	14	10	44	17	24	21	00	28	31	56	53
7	15	20	30	18	25	06	15	29	32	34	58

The Averill Stadiagraph

While considering the general subject of the stadia it may be a matter of general interest to note here that C. K. Averill, of Yonkers, N. Y., has recently introduced a special protractor for the rapid mechanical reduction of notes in plotting stadia topography.

It consists of the major portion of a 10-in. circle graduated into degree-spaces, and a 10-in. radial straight edge divided to 1/10 in. The theory of its construction is based upon the well known formulæ governing the computation of the stadia tables as given in the appendix of this book.

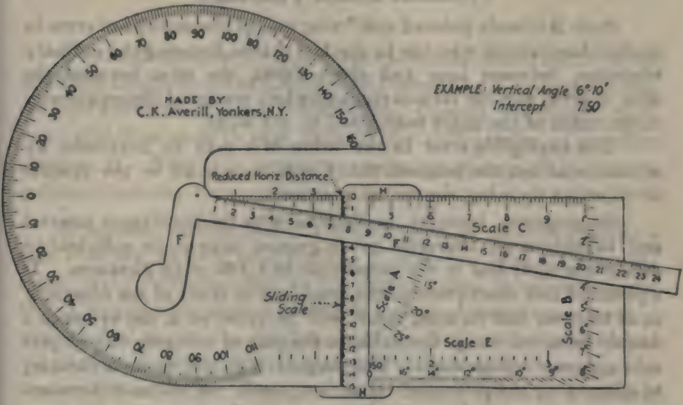

Fig. 67

In the *Eng. News*, Jan. 22, 1914, it says, "The pivoted scale, FF, is the scale of the map and can be changed. This scale is used to determine the horizontal distance from the transit station to the stadia point by setting its graduated edge on the recorded vertical angle reading, on the arc scale A, and moving the sliding scale, HH, until its graduated edge intersects the edge of the pivoted scale, FF, at the recorded rod reading. The intersection of HH and the straight-edge is the point required. No great accuracy is observed in setting the graduated edge of the pivoted scale on the arc scale, A, as the square of the cosines vary but little for a few minutes of angle.

"The graduations of the sliding scale, HH, are determined by the scale of B, which is an assumed scale based on the sines of double the recorded vertical angles, solving the triangle for $\mathbf{h} = \frac{1}{2}$ KI *sin* 2 v. Placing the graduated edge of HH on the rod reading of the straight-edge scale, the intersection of the graduated edge of FF and HH will give the difference in elevation on HH for any

recorded vertical angle setting of FF on the scale B. For vertical angles greater than 20°, scale D is used with a special scale for rod intercepts E.

"The inventor claims an accuracy in plotting horizontal distances equal to the case where the distance is first computed by slide rule and transferred to the map by plotting; and while this would answer for ordinary stadia side shots, it would not seem to eliminate the necessity of computing the traverse lines for closure. The accuracy of elevation is claimed to be 1/8 ft. The Stadiagraph will plot all vertical angles up to 30° and all elevations up to 130 ft. The plotting length for horizontal distances is 10 in."

Prismatic Stadia Telescope

Prof. Richards pointed out * that one of the sources of error in stadia observations was due to the fact that in focusing the ordinary telescope, between long and short sights, the ratio between the wire interval and the various conjugate foci produced inconsistencies in the value of the angle subtended by the wires.

The negligible error in the value of *c* may be overcome by ocular focus but the proposal that *f* or K vary at all in the process of objective focusing is open to question.

He proposed, however, to dispense with the diaphragm interval and substitute an objective prism ground to an angle sufficient to deflect the sight line in the proportion of 1:100 at all distances. If a prism of this description is mounted just in front of the objective so as to cover half its area, there will be received at the eyepiece a double refracted image favorable to the plan of bringing two targets into apparent coincidence. The targets will not form a contact, as effected by the prismatic refraction, until the distance between them is proportional to the distance from the instrument.

The scheme was patented in France under the name of *Diastimometre Sanguet.* The prism was hinged to an objective mount so that it could be thrown into or out of the sight line at will.

The Anallatic Telescope

The optical arrangement by which all observations are automatically corrected to the center of the instrument was the ingenious invention of Prof. Ignaz Porro, of Milan, who put it into practical use in 1823 †

In this type of telescope, the focus of the objective, O, is a constant quantity. Consequently the various rod intervals subtended by the constant angle aFb, are directly proportional to the distance, and the $(c + f)$ constant of the usual system is thereby discarded.

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* *R. H. Richards, M. I. T., in Trans. A. I. M. E., 1891.*

† *La Tacheometrie, ou l'art de Lever les Plans et de Faire les Nivellements, Turin, 1850.*

C being at the center of the instrument, the angle ACB is made equal to the proportion of 1:100, or 34′ 23″, when CA or CB=100 and AB=1. The objective, however, is calculated so that the focus of such rays fall well within the distance CO at F, beyond which they diverge, meeting the anallatic lens at a_1 and b_1.

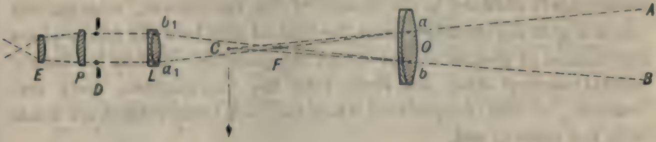

Fig. 68

The anallatic lens, L, is an immovable achromatic combination of a focal length equal to LF=FO and being also a convergent lens, the rays approaching from F are thrown towards the ocular, E P, in parallel lines so as to pass through the stadia wires at the diaphragm, D. This will be so whatever the distance P L, due to racking the ocular in the process of focusing.

It will be apparent that this combination of purpose considerably shortens the focal length of the objective so that the magnification is reduced somewhat, as the anallatic lens is moved toward O. This principle offers a favorable means of adjusting a certain wire interval to suit the conditions of the field instead of attempting to make the stadia wires adjustable according to the generally condemned expedient discussed on p. 107. It stands to reason, however, that the anallatic lens should be fixed at its principal focus from the point F, or the extreme rays, passing out toward the ocular, will not be strictly parallel and the readings will be inconsistent.

For the reason that the focal length of the objective is considerably reduced, the same flatness of field or magnification is not to be expected. Generally, an ocular of very short focus has been employed for this system to restore the power to about ×20 or else the telescope has been increased in length so that the distance FO may be equal to that of a normal objective. The point F being fixed for infinity, the distance FC will gradually decrease as the rod approaches the instrument until a point will be reached when F and C are coincident. The minimum range may be determined by experiment, but it is to be understood that the anallatic method, as applied to instruments of average size, is generally unreliable under a 50-ft. or 16-m. range.

Perhaps the strongest argument advanced against the stadia as in favor of the gradienter screw is the fact that all gradienter observations are taken with the central horizontal cross wire and immediately reduced to the center of the instrument, whereas the stadia sights are removed from the optical axis into the zones where aberrations of greater or less extent are possible. While the instrumental constant does not enter into the calculations, they are, however, no less complicated when inclined observations are made with the vertical rod.

Fig. 69

Its invention, as an attachment to the Theodolite for gradient and telemeter work, is usually attributed to Prof. S. Stampfer, of Vienna, in the early seventies of the last century, but the micrometer principle certainly was not new at that date.

Considering the variety of useful purposes to which it may be applied, it has not yet achieved the popularity it merits. It can be attached to either the vertical or horizontal tangent movements, can

be used for observing or establishing gradients, for determining distances with either a fixed or variable rod interval, or for measuring small angles without the use of the graduated limb.

Theory

The principle of its construction is very simple. If the radius of the clamp which controls the movement of a sight line is divided into 100 parts and this value used as the pitch of the thread for the tangent screw, it is evident that one revolution of the screw will communicate a movement in the sight line equal to an angle of 1% or 34′ 23″. In other words, the tangent of this angle is equal to 1 ft. at 100 ft. from the center of the instrument, 2 ft. at 200, etc.* The length of clamp which we use would make such a pitch too coarse for ordinary requirements so that, by preference, we take 1-200th of the clamp radius and turn the screw two revolutions for each 1% to accomplish the purpose.

If the tangent screw of the transit telescope is provided with a drum head that is divided into 50 parts, each division will then represent .01% or will encompass .01 of any unit of measurement on a rod at 100 such units from the center of the instrument. Just above the graduated drum is an indicator scale of spaces equal to the pitch of the thread, merely to measure the number of whole revolutions, the fractional part being taken directly from the drum.

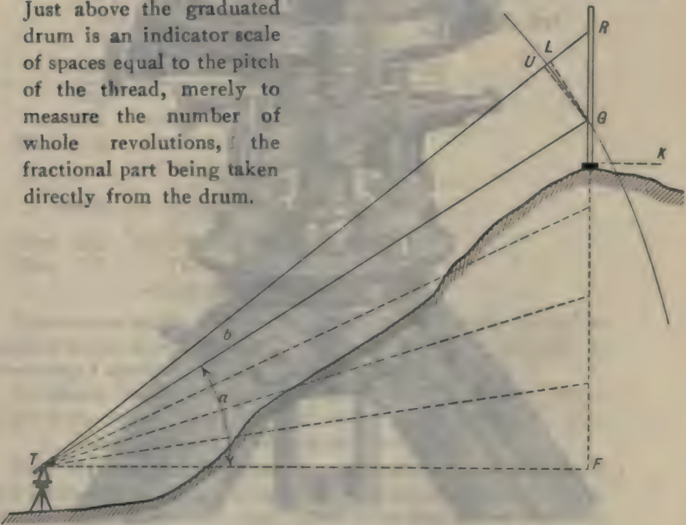

Fig. 70

* *Engineering and Surveying Instruments, I. O. Baker, 1906, p. 209.*

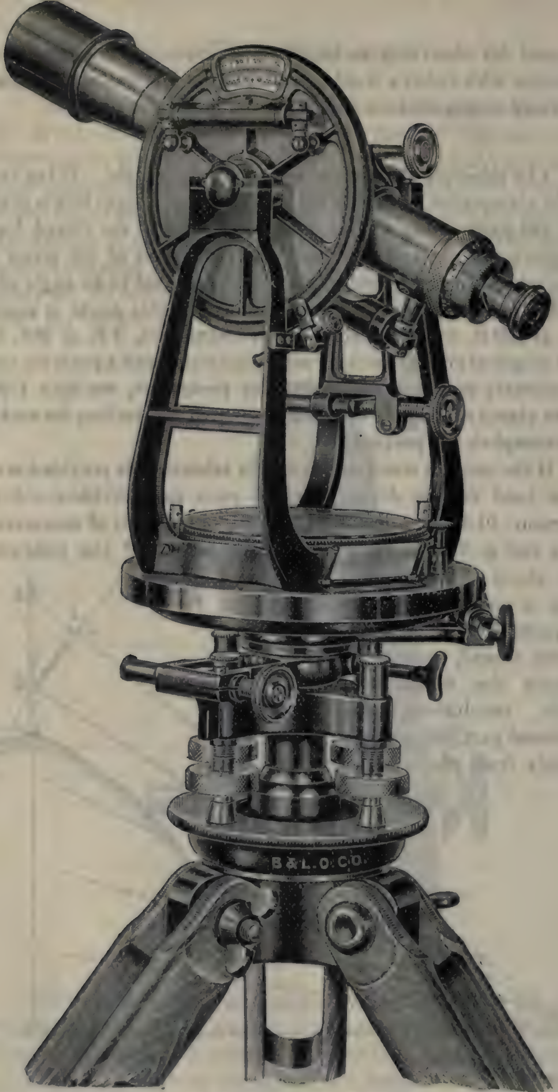

Fig. 71—Showing General Equipment with Vertical Clamp, Telescope Bubble and Verticle Circle with Closed Cover Guard (open if desired) and one Double Vernier as covered under the No. 45 4½ in., No. 55 5 in. and No. 65 6 in. Tachymeter.

The clamping arm of the gradienter, described in connection with the alidades on page 57, however, is short enough so that the pitch of the thread can be cut 1:100 and the drum divided into 100 parts. Some transit instruments have been constructed with half length tangent clamps for the purpose just explained, but the plan is neither necessary nor desirable.

ADJUSTMENT

The gradienter drum is held in any required initial position by a circular spring and clamping nut. If it is desired to begin with the zero at the index for any particular position of the telescope, grasp the knurled head with one hand, so that the screw may not turn, and revolve the graduated drum on the shank until the desired relationship has been secured.

To measure distances with the gradienter, level the telescope, set the index to zero as above and turn a certain number of revolutions, up or down, until the central cross wire strikes the rod. Turn two more revolutions and read the intercept for use in the subsequent calculation.

Let UG be perpendicular to TR and let LG be perpendicular to TG. Let (n—1) revolutions equal **a** and let the angular value of the last revolution be equal to **b**. Then

$$\text{UGR} = \mathbf{a} + \mathbf{b} \text{ and } \cos(\mathbf{a}+\mathbf{b}) = \text{GU} \div \text{GR}$$

$$\text{LGR} = \text{GTF} = \mathbf{a}$$

$$\text{LRG} = 90° - (\mathbf{a}+\mathbf{b}) \text{ and } \text{RLG} = 90° + \mathbf{b}, \text{ but}$$

$$\text{LG} : \text{RG} :: \sin \text{LRG} : \sin \text{RLG}, \text{ then}$$

$$\frac{\text{LG}}{\text{RG}} = \frac{(\cos \mathbf{a} \cos \mathbf{b}) - (\sin \mathbf{a} \sin \mathbf{b})}{\cos \mathbf{b}}$$

$$\text{GL} = \text{GR} \cos \mathbf{a} - (\sin \mathbf{a} \tan \mathbf{b})$$

$$\text{Dist. TG} = 100\ \text{GL}$$

$$= 100\ \text{GR} \cos \mathbf{a} - (\sin \mathbf{a} \tan \mathbf{b})$$

$$\tan \mathbf{b} = \frac{\text{LG}}{\text{TG}} = \frac{1}{100}; \text{ substituting,}$$

$$\text{Dist. TG} = \text{GR}\,(100 \cos \mathbf{a} - \sin \mathbf{a})$$

$$\text{Dist. TF} = \text{TG} \cos \mathbf{a}$$

$$= \text{GR}\,(100 \cos^2 \mathbf{a} - \tfrac{1}{2} \sin 2\mathbf{a})$$

If **a** were an angle of depression, the vertical angle should be measured to the upper leg of the **n**th revolution. The angle **b**, representing 1 double revolution of the screw, is 34′ 23″.

To obtain the vertical coördinate, FG, let KG = H.I. Measure one more complete revolution of the gradienter above G, then,

$$\text{Dist. FG} = \text{GR}\,(100 \cos \mathbf{a} \sin \mathbf{a} - \sin^2 \mathbf{a}).$$

One of the principal objections with reference to the accuracy of the gradienter is the conceivable lost motion, or back lash, in the screw incident to continued wear. Our screw shanks are of German silver operating in a bronze encasement. While the pitch of the thread remains unaltered, the operation of the screw, ought to be accurate. To take up lost motion we provide a set-screw, as shown

in the lug of Fig. 69, that can be tightened from time to time as the occasion requires. Beyond a comparatively short interval each side of the horizon, however, the gradienter ceases to be accurate because the pitch of the screw gradually looses its strict proportion with the chord of the arc it is supposed to measure.

The discussion on the gradienter, pp. 15 and 24, does not profess to take vertical angles into consideration while working close to the horizon.

Hypsometric Leveling

This practice with the gradienter consists of determining a vertical angle in terms of its tangent and having measured the inclined distance with the tape, or with the gradienter itself, to find the difference in elevation by multiplying the measured distance by the sine of the angle found.

For instance, suppose the gradienter to be turned 3 revolutions and 18 spaces below the horizon in order to strike a certain station peg. This movement is equal to 168 spaces as previously explained, or a gradient of 1.68%. Referring to table, p. XXVI, we find this is equal to an angle of 57′ 45″, the natuıal sine of which can be used to get the difference in elevation.

Tables compiled from the *Army Manual* may also be used for this purpose. Having found the percentage equivalent in degrees, simply add the multiple parts as when using an ordinary traverse table. (See p. XXVII Apx.)

Gradients

Let it be required to run on a gradient of 2% compensated to 1.94%. Turn the drum three complete revolutions and 44 additional spaces either in elevation or depression. Take the H. I. and observe the rod at the several successive stations. If H. I. = 5, and the rod reading is 4, a cut of 1 ft. would be implied. If the rod reading is 6, a fill of 1 ft. will be implied, etc. This is the most facile method known for running grades in highway or railroad construction. (See also page 24.)

To Measure Horizontal Deflections with the Gradienter

Contributed by Prof. D. C. Humphreys, Washington and Lee Univ., Lexington, Va.

When the compass was good enough for land surveys, a tree that happened to intercept an alignment was good enough for a rod and the survey proceeded by setting up on the other side and continuing the line on the same magnetic bearing. Now, however,

he error must be kept within two feet per mile, or 1′ 18″. If small angles of deflection, in passing trees, could be measured with precision, there is no reason why an alignment could not be preserved with great accuracy.

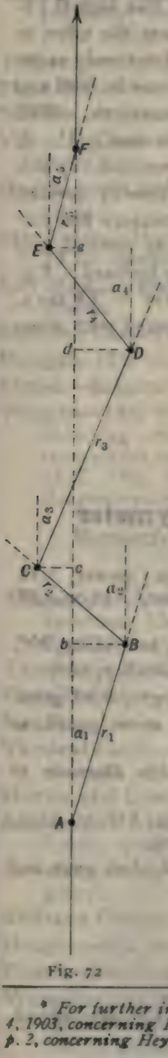

Fig. 72

The horizontal circle should be provided with a gradienter, and for this class of work the circle should remain clamped. Whenever the gradienter was at the zero of its scale, the direction would then be secured.*

The gradienter drum for the horizontal circle should be graduated to read in radians, which requires only that, as usual, the pitch of the thread shall be 1/100 of the radius of the clamp and that the drum shall be adjustable and divided decimally.

To run a straight line through a forest without cutting large trees:—

The distance $Bb = r_1 \sin a_1$,

$$Cc = r_1 \sin a_1 + r_2 \sin a_2, \text{ and}$$

$$Dd = r_1 \sin a_1 + r_3 \sin a_2 + r_3 \sin a_3$$

or generally, the amount of departure from alignment is $\Sigma\, r \sin a$

At the point F, where it is desired to resume the straight line,

$$\Sigma_0^5 \,(r \sin a) = 0, \text{ or}$$

$$\Sigma_0^4\, r \sin a + r_5 \sin a_5 = 0$$

The last equation enables the surveyor to assume either r_5 or a_5 and to calculate the other. Assuming a_5, then

$$r_5 = -\frac{\Sigma_0^4\, r \sin a}{\sin a_5}$$ or, if all angles are quite small, approximately,

$$r_5 = -\frac{\Sigma_0^4\, r a' \sin 1'}{a'_5 \,.\, \sin 1'} = -\frac{\Sigma_0^4\, r a'}{a'_5}$$

So far as the final result is concerned, it is about as convenient to measure a in minutes as in radians but it is sometimes desirable to drive stakes on the true line, in which case the deflections in radians are more convenient.

* *For further investigation of this practice, "Engineering", London, Dec. 4, 1903, concerning Fergusson's Percentage Theodolite and Zeit. fur Inst., 1905, p. 2, concerning Heyde's Zahnkreiss Theodolite.*

"Usually it will only be necessary to use two deflected lines to pass a tree or other obstruction, by deflecting 1% to the right followed immediately by an equal deflection to the left, measuring equal distances, on the two lines. If the deflected line departs 1% from the true direction, it is much easier to compute the offset to the line than if the deflection be 30′ (0.874%); and not only easier but more accurate, for the least count of the gradienter is .0001 and by estimation 0.1 of this making the reading accurate to .00001 or 0′.034, whereas the least count of the vernier is usually 1′. It may be said that the accuracy of the gradienter method, by this comparison, is about thirty times as great as deflection by vernier.

"With the azimuth gradienter it is possible to measure horizontal distances if the rod is held horizontal instead of vertical, and no correction has to be made if the sight should be inclined. Fig. 70 (supposing GL horizontal) TG = 100 GL and GF = TF *tan* a. The simplicity of these formulæ is apparent compared with those given above."

Specifications

Bausch & Lomb 4½-inch Tachymeter

"Mountain"

Telescope—Erecting, 8 in. long; aperture, 1⅛ in.; power, ×18. Inverting, 8¾ in. long; aperture, 1 in.; power, ×16 or ×20. Both types transit at both ends. Cryptic focus.

Telescope Bubble—3½ in. long, 2mm divisions, sensibility, 30″.

Plate Bubbles—1½ in. long, 2mm divisions, sensibility, 60″.

Needle—3 in. long, tubular, sapphire mount; weight, 0.50 grms.

Vertical Limb—2¼-in. radius, open or closed cover guard, as desired.

Horizontal Limb—2⅜-in. radius, 5½-in. outside diameter of plates.

Standards—Aluminum bronze, bell metal journals; 5½ in. high, 2⅝-in. spread; 3⅝-in. diameter of compass ring.

Leveling Screws—German silver, 1⅛-in. head, dust caps and guards.

Shifting Center—½-in. range.

Height—11¼ in. with circle.

Finish—Bakelite yellow and Kahki pyrolin.

Weight—9½ lbs.; in case with accessories, 14 lbs.
Split leg tripod, No. 74, 57-in. legs, 7¼ lbs. Ext. leg tripod, No. 74-X, 7½ lbs.

Bausch & Lomb 5-inch Tachymeter

"Engineers"

Telescope—Erecting, 11 in. long; aperture, 1¼ in.; power, ×22. Inverting, 10¾ in. long; aperture 1¼ in.; power, × 20 or ×30. Both types transit at both ends. Cryptic focus.

Telescope Bubble—4¼ in. long, 2mm divisions, sensibility, 25″.

Plate Bubbles—2 in. long, 2mm divisions, sensibility, 50″.

Needle—3⅞ in. long, tubular, sapphire mount; weight 0.64 grms.

Vertical Limb—2½-in. radius, closed or open cover guard as desired.

Horizontal Limb—2¾-in. radius, 6½-in. outside diameter of plates.

Standards—Aluminum bronze, bell metal journals; 6½ in. high, 3¼-in. spread; 4⅞ in. diameter of compass ring.

Leveling Screws—German silver, 1¼-in. heads; dust caps and guards.

Shifting Center—½-in. range.

Height—12¼ in. with arc; 13¾ in. with circle.

Finish—Bakelite yellow and Kahki pyrolin.

Weight—13½ lbs.; in case with accessories, 25 lbs.
Split leg tripod No. 75, 60-in. legs, 9½ lbs.; can be reduced for city surveys when requested. Ext. leg tripod No. 75-X, 10¾ lbs. Heavier tripods listed below furnished if the preference is so stated.

Bausch & Lomb 6-inch Tachymeter

"Cadastral"

Telescope—Erecting, 12¼ in. long; aperture, 1⅝ in.; power, ×24. Inverting, 12 in. long; aperture, 1⅝ in.; power, ×23 or ×32. Both types transit at eye end only. Cryptic focus.

Telescope Bubble—4½ in. long, 2mm divisions, sensibility, 20″.

Plate Bubbles—2 in. long, 2mm divisions, sensibility, 50″.

Needle—3⅞ in. long, tubular, sapphire mount; weight, 0.64 grms.

Vertical Limb—2½-in. radius, open or closed cover guard, as desired.

Horizontal Limb—3⅛ in. radius, 7-in. outside diameter of plates.

Standards—Aluminum bronze, bell metal journals; 6¾ in. high, 3¼-in. spread; 4⅞-in. diameter of compass ring.

Leveling Screws—German silver, 1¼-in. heads, dust caps and guards.

Shifting Center—9/16-in. range.

Height—12¾ in. with arc, 14½ in. with circle.

Finish—Bakelite yellow and Kahki pyrolin.

Weight—16½ lbs.; in case with accessories, 25½ lbs.
Split leg tripod, No. 76, 60-in. legs, 11½ lbs. Ext. leg tripod, No. 76-X, 13 lbs.

Fig. 73
Page 128

THE THEODOLITE

For geodetic, cadastral and mining surveying, or primary and secondary triangulation, for latitude and time observations and other classes of field astronomy, and for a variety of other high class work, where the most substantial and accurate construction is required, the transit-theodolite is the instrument preferred and adopted.

The absence of the compass permits casting the standards in a single solid U-shaped structure with its broad base mounted directly over the flange of the vernier plate. This method gives the instrument a rigidity impossible in other types and the design provides for a most successful and symmetrical combination, uniting strength with beauty of form.

The size of theodolites and all transit instruments is technically fixed, both in Europe and in America by the outside diameter of the divided circle. The 4½-inch, the 5-inch and the 6-inch models are of the so-called portable size that are carried about in a single case. They are all designed for excellent service and hard use and may be mounted either on the regular 4-screw base or upon our own 3-screw model, at the discretion of the purchaser. At the end of this chapter the respective weights are given in the specification tables.

The 7-, 8-, 10- and 12-inch models will not be carefully considered in this edition. The European 3-screw base and beveled horizontal plates habitually furnished with these instruments are not yet popularly adopted for the smaller models. Fig. 73 on the opposite page shows a No. 456 4½-inch model mounted on the European 3-screw base, as we have made them occasionally for Gov't. Engr's. operating in remote districts. The telescope is provided with the cryptic focus and a magnification of $\times 18$ if erecting, or either $\times 16$ or $\times 20$ if inverting. The circles read to $30''$, and the weight of the entire equipment with extension leg tripod does not exceed 9 kg.

Open Telescope Bearings and the solid U-standards are the two principal features which differentiate the theodolite from all other instruments. Some of the smaller sizes are constructed with closed bearings as in Fig. 74, but these are for use on general work where the most compact construction is preferred and where the compass is not regarded as an indispensable adjunct.

In our lectures on adjustments under IX, p. 72, we emphasize the necessity of concentric mounting in the objective, straight-line movement in the focusing device, a telescope whose geometrical and optical axes are coincident and mid-way between the bearings, and finally, journals that are spaced equi-distant from the center of the vertical axis of rotation. Not all makers have been capable of

meeting these requirements, and geodetic engineers began long ago to correct for residual collimation error, due to eccentricity of the telescope, by taking two double sets of readings: first, with the telescope erect, then inverted, then by removing it from its bearings and, turning the instrument about, duplicate the system of observations with the telescope thus reversed.

This implies the necessity for readily accessible bearings, as shown in Fig. 79. In this case the axle hubs are always cylindrical, resting in V-bearings or interrupted cylindrical surfaces. The caps are raised or opened, the plungers of the opposite tangent screws are released by winding up the little set screws at the end of each spring house (see Fig. 81) when the telescope, with circle and tangent screws, can be picked out of the standards.

The nose-pieces at each side, against which the tangent screws operate, must be interchangeable in fit. The illustration on p. 128 will also suggest the necessity for this. The tangent adjustment, which is attached to the vertical circle, is used at will to center the Control Bubble as with the Index Adjuster described on p. 96.

Of the various requirements enumerated in the last paragraph on page 129, the one which provides for bearings that are equally spaced from the vertical axis of rotation, is dependent for its permanency upon the capacity of the standards to resist ordinary shocks and lateral strains. That the U-standards of the theodolite are best adapted to preserve collimation for all distances is a proposal that need not be submitted for debate.

The ordinary truss standard of the ordinary transit, that is shaped like the letter "A", must rely upon screws at the contact point from the under side for its stability, and the hazard of ordinary transportation, particularly in carrying cases with cupboard doors, is sufficient to cause derangements that will come into existence without being noticed.

The standards of our Transits are reinforced at the contact with the base plate with this purpose in view. It is not a mere novelty in construction. The remarkable stability will be apparent by grasping the journals—one in each hand—and testing for side-play or lost motion. This is a consideration of fundamental importance that has never before received adequate treatment.

The consolidated U-standard of our Tachymeters is the most substantial construction that can be devised in one piece and still contain a compass box. Eventually, when the American surveyor comes to depend less upon the needle and will be satisfied with those which are shorter, a still more substantial and elegant standard can be designed.

All standards of unsymmetrical design which violate the mechanical sense, offend the artistic sense and transgress the common sense are diametrically opposed to the ethics of high class construction.

The Striding Level

This accessory is valuable in securing and maintaining ideal conditions in the telescope axis. In many theodolites the standards have been made so low, in favor of stability, that the telescope could not be reversed in altitude except by removing it temporarily from its bearings and replacing it either in an opposite or inverted position. For this purpose the telescope bearings must be open and easily accessible, not only for the ready application of the striding level to the pivots but for the purpose of collimation and centering tests just explained.

The axis hubs must be perfect cylinders and of equal diameter or the line of sight will describe curves in altitude. One of the important uses of the striding level is to test this equality of bearings precisely as described for collar disparity in wye levels on p. 14. Recesses, indentations or saw-tooth ridges in the axis bearings, such as have been utilized to give increased stability to an imperfectly designed standard, would not be permissible for this work. In such bearings there are multiple surfaces and seatings to reconcile. The astronomer or geodecist would not utilize such telescope bearings because he knows that the cylindrical hub is the only one in which friction is minimized and the only one that will remain in adjustment with any persistency. (See p. 77.)

Adjustment

It must be assumed that the telescope hubs are perfect cylinders and that the contact points of the V-notches will always find a seating at the same place. With the telescope horizontal, open the bearings and place striding level on the pivots; center bubble with leveling base, then pick up striding level and turn end-for-end. If the bubble does not remain centered, correct half of the error in the leveling base and the other half with the adjusting screws of the vial tube. Reverse again and, if there continues to be a slight deviation, correct half errors as before and repeat until no divergence exists.

In reality, this process adjusts the bubble axis to parallelism with the contact point of the notches, and incidently it brings the contact surfaces of the telescope axis into a truly horizontal position; but there is no assurance yet that it will remain horizontal when reversed on the vertical axis.

To Adjust the Telescope Axis, therefore, swing the instrument 180° on the vertical axis. If the bubble runs off center, correct half the error with the leveling screws and the other half in the adjustable bearing block in one of the journals. Swing back 180° and continue to rectify discrepancies by this means until the

bubble remains centered in both positions. Now swing 90° and simply bring the bubble to the center of its scale with the other set of leveling screws. When using the 3-screw base, first swing the striding level parallel to any two of the leveling screws, and in the final step swing the longer axis of the vial over the third screw and use that screw alone for centering purposes. This process adjusts the vertical axis to perfect verticality and at the same time has made the telescope axis horizontal in all positions.

Fig. 74—Showing general equipment with Vertical Clamp, Telescope Bubble, Vertical Circle (open or closed guard as desired,) with one Double Vernier; covered under the No. 455 4½-in., the No. 505 5-in. and the No. 605 6-in. Theodolite.

Note: For the method of closing this type of vertical circle against dust or rain, see Fig. 71, p. 122.

The striding level in such a case supercedes the transverse plate bubble which may now be brought to the center of its scale if it happens to be out of adjustment. In fact, if one had broken both his plate bubbles, he might get along very nicely if he had only a striding level and followed the above suggestions. Usually

the striding level is not less sensitive than 20″ by which 5″ can be estimated by quarter divisions. It is important in straight line work and in surveys where angles are being read between points in widely separated horizontal planes.

To Test the Side Adjustment, rock the striding level back and forth over short arcs either side of the vertical. If it runs to the right at one side and to the left at the other, it shows that the vial lies diagonally across the line of the horizontal axis. The rectification is identical with that of the "wind adjustment" in the wye level which may be consulted on p. 8. A striding level should at all times rest without strain or interference on its bearings. This accounts for looseness of contact in this construction. To prevent damage by wind or accident, however, a pin is provided in one of the standards that engages an eye in the leg of the level mount.

To Test the Equality of Pivots, remove the telescope from its bearings and replace it after the instrument has been reversed. If the vertical axis is truly vertical and if the bubble has been accurately adjusted to the V-notches and now shows a displacement, the axles of the telescope may be suspected of inequalities. The effects of even slight changes in temperature on the whole instrument should be carefully considered and weighed in the evidence. It is not so easy to keep a large theodolite in perfect equilibrium even in apparently quiescent atmosphere. If the axle hubs are actually shown to be in disparity by this test, the bubble will rise toward the larger one and the matter should be either submitted to the precise mechanic or double observations should be made in conformity with accredited practice.

Applied to Mining or Tunnel Surveys, the striding level cannot reasonably be expected to control the auxiliary telescope whose inevitable errors must be corrected either by interchangeability or double reversed observations. Steep sights, of relatively great vertical and short horizontal components, will magnify small instrumental errors unless controlled by a striding level. For straight line work with the main telescope, on inclined sights, a properly adjusted striding level will therefore be conducive to the best results; but for tunnel alignment, or other such work in the plane of the instrument, double reversed sights, as in the collimation test, is the infallible measure.

The striding level that is mounted on two inside bearings on the axle of the telescope is an expedient of very doubtful benefit. If the inside bearings cannot be reconciled to the pivots on which the telescope revolves, the idea must be regarded as a fallacy. Theoretically the idea is just as absurd as the use of collars on alidades (see p. 55).

The Beveled Horizontal Limb

All of the larger Theodolites and Alt-azimuths, including the 7-, 8-, 10- and 12-inch models, have been habitually made with beveled plates and supplied with the European 3-screw base. Some manufacturers have argued laboriously against the adoption of the beveled limb for every conceivable reason except the paramount issue which involves increased cost of manufacture.

It may be consistently contended that in looking straight down on the line of separation between vernier and limb, when the reading edge is worn, one may not make quite so careful an estimate of a coincidence as when looking toward the scales at an angle of say 60°. On the other hand, the slanting line of vision against a flat limb is what offers the opportunity for parallax in reading when the fit becomes imperfect by abrasion.

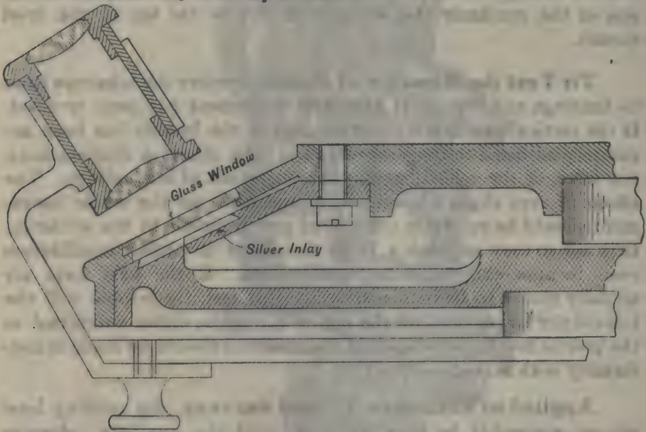

Fig. 75

There can be no doubt that the beveled limb is more readily accessible and that the eye can be brought near to it without interfering with other parts of the instrument. This is especially true of the more finely divided circles that are supplied with powerful attached magnifiers. There is no reason known to us why the bevel limb cannot be just as accurately and durably made but it costs more to build than the flat limb. Diagrams are always of technical interest if they are not purposely calculated to mislead the reader. The diagram shown in Fig. 75 represents the latest method of applying this principle to the 6-inch Theodolite, as illustrated in Fig. 77.

The beveled horizontal limb offers the best arrangement for the adoption of the **Attached Magnifiers** or fixed reading glasses, as they are sometimes called. These are an adaptation of the

Ramsden ocular which gives a positive image of the field. The lines and figures appear in their proper relation—erect as it were. The reason this combination of lenses shows objects inverted in a telescope is because the image of the field is projected beyond the plane of the diaphragm in an inverted position by the objective. The drawing shows the magnifier mounted nearly normal to the bevel of the limb, but some engineers prefer to have the magnifiers slant a little (as we are required to build them for a vertical circle, or a flat limb) in order to sight obliquely along the lines in selecting the position of coincidence. The objection to the plan is that the upper and lower portions of the field are not in perfect focus at the same time, but prismatic cover glasses would, in a great measure, remedy this.

Our magnifiers are mounted on radial arms that move in short arcs following the curvature of the reading edge as shown in Figs. 73, 77 and 81. The method of mounting magnifiers for the vertical limb is also shown in these illustrations.

The Steinheil Heliotrope

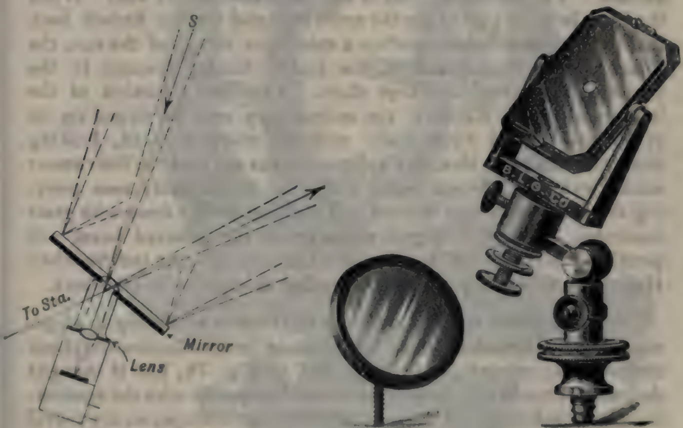

Fig. 76

In surveys limited in area, even to the confines of municipal boundaries, the atmosphere is frequently so dense that ordinary targets are either indistinct or invisible. In all such cases the heliotrope is a serviceable accessory for picking up a sunbeam and projecting it into the theodolite precisely as though the target were a luminous object.

Otherwise the heliotrope is not much used for distances under 10 miles for the heliotroper must be expert and attentive, and unless

the size of the mirror is regulated to accord with the length of sight, the image becomes too large and brilliant. The size of the image, however, can be, and ought to be, regulated to suit the distance in about the proportion of 1 inch per 20 miles. Our mirrors are 1⅛ inches across the shorter diameter. This would be ample for 25 miles. As the distance decreases, cardboard stops can be cut out and pasted over the outer edge of the mirror.

This precaution is taken to reduce the dazzling effect which could otherwise be modified for city work with a sun glass shutter, as illustrated in Fig. 89. The beam from the sun should be regarded as a cone of light coming to a point at the center of the mirror. The sun's diameter is about 32′, so that the projected beam will have the same angular value. This is sufficient to subtend about 50 feet per mile, or about 1000 feet in 20 miles. No very accurate adjustment for centering on the theodolite station is necessary.

The heliotroper has only to mount the apparatus over the center of the monument. In the top of the tube supporting the mirror is a small lens, and in the bottom of the tube is a white reflecting surface. The heliotroper turns the tube toward the sun. When the little lens has picked up the sun's rays and there is flashed back from the white reflecting surface a miniature image of the sun, the image should be centered in the small circular opening in the center of the mirror and kept there. The lower portion of the instrument being clamped, the mirror may now be tipped on its pivots and turned equatorially with its standard until, looking through the little orifice, the heliotroper has covered the instrument station with the reflected beam, which can be distinctly seen piercing the atmosphere out beyond. This need not be done with great exactness, for the divergent character of the reflected beam will permit an error of 15′ in setting. On this account the Steinheil Heliotrope should rightfully be regarded as superior to all others.

It is generally screwed into any convenient mounting in alignment back of the instrument to facilitate intercomunication from the observation station, as suggested in Fig. 76; but if thought desirable it could also be mounted interchangeably with the equatorial solar attachment and the auxiliary mining telescope, on the swivel mount, as shown in Fig. 77 also in Figs. 98 and 106. By this means signals can be exchanged either by the Morse code or by a prearranged system. When each station is provided with a heliotrope, they find each other readily by slowly swinging the sunbeam in the general known direction. Fig. 77 shows one of our latest designs in City Triangulation and Tunnel Theodolites with the heliotrope mounted detachably. Fig. 76 also shows the **supplementary mirror** which is screwed into some substantial mounting and used only when the signal flash is to be projected onward in the same general direction as the original sun beam.

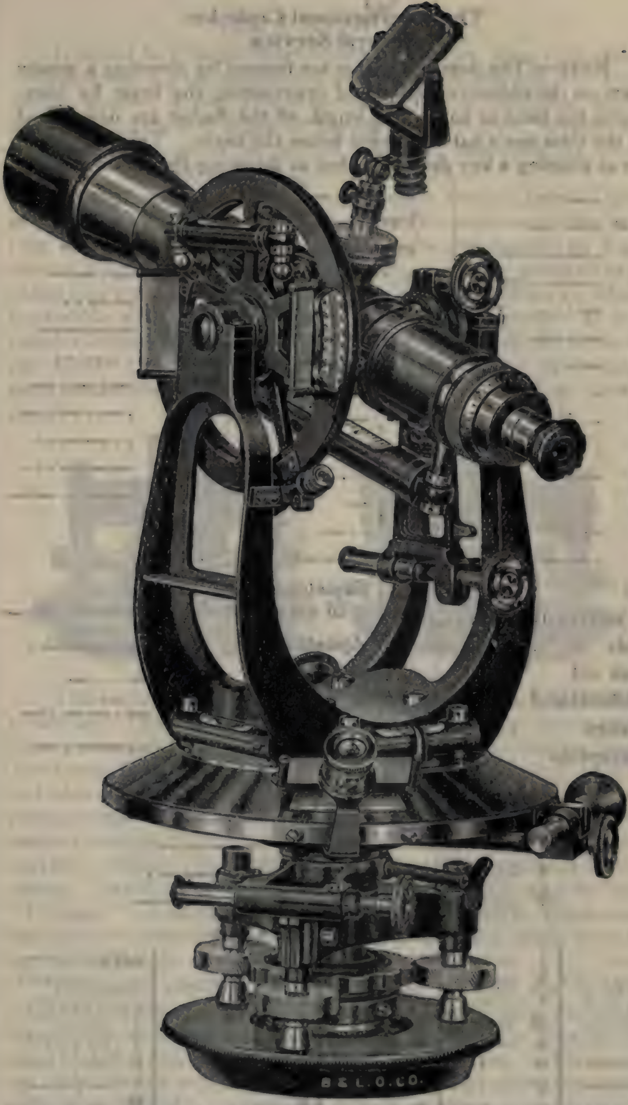

Fig. 77– Showing General Equipment with Vertical Clamp, Telescope Bubble, Vertical Circle (open or closed guard as desired) and Double Opposite Verniers, as covered under the No. 456 4½-in., No. 506 5-in. and No. 606 6-in. Theodolites.

The International Code for General Service

NOTE:—The dots and dashes are formed by directing a steady beam on the objective station and interrupting the beam by interposing the hand or hat. The length of the flashes are determined by the time one's hat is lowered below the beam. Telegraph then, not as pressing a key downward but as thrusting the hat downward.

a	.—	k	—.—	v	...—
ã æ	.—.—	l	.—..	w	.——
à	.——.—	m	——	x	—..—
å	.——.—	n	—.	y	—.——
b	—...	ñ	——.——	z	——..
c	—.—.	o	———	ch	————
d	—..	ö œ	———.	.	......
e	.	p	.——.	;	—.—.—.
é	..—..	q	——.—	,	.—.—.—
f	..—.	r	.—.	?	..——..
g	——.	s	...	:	———...
h	....	t	—	!	——..——
i	..	u	..—		
j	.———	ü ue	..——		

Wait	.—...	Divisional bar of a fraction	——————
Are you ready	..—.—	Apostrophe	.————.
Ready	—.—	Hyphen	—...—
Break off	—...—	Bracket	—.——.—
I understand	———	Underlining	..——.—
Finished	.—.—.		
Interruption	....		

1	.————	6	—....
2	..———	7	——...
3	...——	8	———..
4	....—	9	————.
5	.....	0	—————

American Morse Code

a	.—	j	—.—.	s	...	1	.——.	zero	———
b	—...	k	—.—	t	—	2	..—..	.	..——..
c	.. .	l	———	u	..—	3	...—.	,	.—.—
d	—..	m	——	v	...—	4	....—	:	—.— . .
e	.	n	—.	w	.——	5	———	;	... ..
f	.—.	o	. .	x	.—..	6	......	?	—..—.
g	——.	p	.....	y	.. ..	7	——..	!	———.
h	....	q	..—.	z	... .	8	—....	¶	————
i	..	r	. ..	&	. ...	9	—..—	$	... .—..

The 3-Screw Base

It would be difficult to say where the 4-screw base originated. It would be a safe speculation to say that it was the first slow motion applied to the ball-and-socket joint with compasses, astrolabes and planispheres in use in Europe prior to 1700, because the ball-and-socket cannot be leveled in a rigid mounting by three screws and only two hands with which to work.

We shall probably never depart from the 4-screw base so long as the ball joint is retained. Many of the more conservative adhere to it by reason of its so called rigidity—secured mainly by high tension in the screws which, in turn, impart an unequal strain on each of the four arms. This is especially evident in wye levels in which the bubble will move off center with no apparent cause. It is a practical impossibility to equalize the strain in each of the arms of the 4-screw head. A slight change in temperature accentuates these conditions and causes a binding known in England as "headache."

Fig. 78

The 4-screw base is never used on any other type of scientific apparatus. The 3-screw base is an ideal construction because the centers are never distorted or strained by lateral pressure or jambing. The longer arms permit better leverage and more sensitive control and with numb fingers, in cold weather, the improvement is very pronounced. The Germans and French have used the general design shown in Fig. 73 which is, no doubt, best suited to the heavier instruments that will not respond to slight easements in the spring. The English overcame the necessity for a spring in their "Tribrach" in which each leveling screw terminates in a spherical base that provides for a universal seating as well as a means of locking the instrument to the lower plate.

The special tripod heads necessary with all these models have made the introduction of the 3-screw principle a very deliberate matter until we introduced the patented model shown in Figs. 8, 38, and 77 when a new impetus was created in its favor. This model not only fits the ordinary tripod but is provided with the usual shifting center and a knurled ring to lock it against easement while carrying it on the shoulder. When setting up, the knurled ring

should always be loosened. When the instrument has been leveled, however, it may be set down, not too tightly, against the ring, merely as a precautionary measure. The plate bubbles may still be controlled within one or two divisions. (See I, p. 62, also p. 18).

The Declinatoire

For much of the ordinary class of surveying, engineers are beginning to feel that while a more serviceable instrument than the compass has never been devised for rapid and approximate orientation, still it has occupied a too conspicuous place, considering its

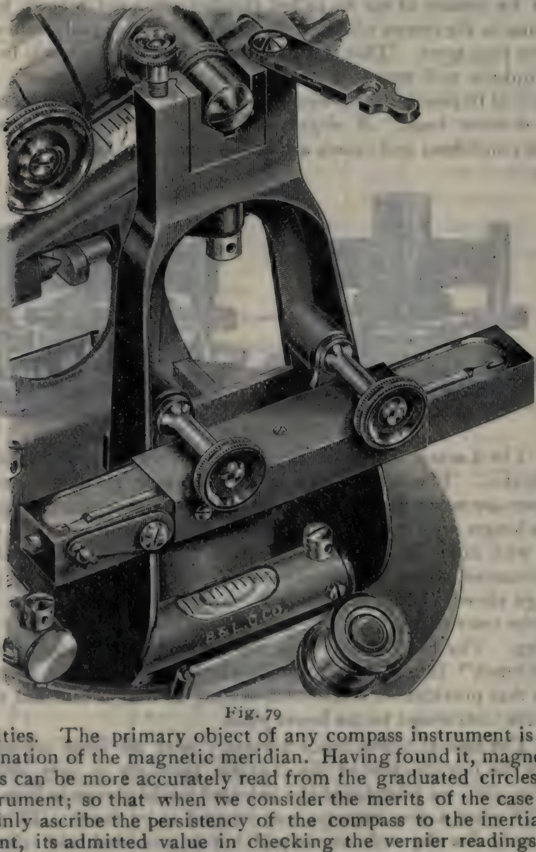

Fig. 79

capabilities. The primary object of any compass instrument is the determination of the magnetic meridian. Having found it, magnetic bearings can be more accurately read from the graduated circles of the instrument; so that when we consider the merits of the case we can mainly ascribe the persistency of the compass to the inertia of precedent, its admitted value in checking the vernier readings of careless operators, for ruuning rough traverse over cheap property, and preliminay lines in re-location.

Whenever a magnetic meridian is required, it may be laid off by attaching a Declinatoire to the side of the standard as shown in Fig. 79. With the circles set to zero, the instrument can be maneuvered on the azimuth axis with the lower tangent screw until the needle floats freely in line with the indices. Open the upper clamp and, directing the telescope to the objective point, the bearing of any course can then be read directly to the nearest minute from the graduated circles.

The manufacturer is required to set the N-S line of the compass in the same vertical plane with the transit telescope and is also required to set the index line of the Declinatoire so that it will be parallel with the line of collimation.

Adjustment

This can be accomplished only by placing the theodolite at one end of a previously determined solar or stellar meridian. With the vernier set at zero, the telescope is set in the meridian. The needle's declination at the point of observation must be known (see p. 90). Lay this off to the E. or W., as the case may be. Turn the telescope in that direction and measure the amount on the vernier plates. Unclamp the needle and see if it swings freely in alignment with the indices at the end of the box. If not, it will be permissible to remove the set screws and rub down one of the bearings with a piece of emery paper. By repeated trials a satisfactory fitting and adjustment may be accomplished.

We give these instructions in case an instrument should receive a blow and thus derange the relationship secured between the attachment and the standards. The instruments used by the Can. Top. Survey are regularly tested at the Dominion Magnetic Observatory where it has been decided that the amount of the index error is likely to change, not only from accident, but from weak standards, improper packing, etc., and that on account of the diurnal variation (see p. 90) a perfect adjustment within 5′ or 10′ is not possible.

If the adjustment as suggested above is not undertaken, the constant error of deviation should be known and applied to each magnetic observation where actual values are sought. In the case of Alidades it is a matter of only relative or scant importance to have the magnetic axis parallel, or coincident, with the collimation axis; for a constant error applied to each observation will not destroy the proportion of the traverse.

To re-balance the needle, for different latitudes, proceed as directed on p. 48. Handle the needle with great care and do not return it to the pivot until it has been wiped dry from the moisture of the fingers. The frequent shifting of the counter weight by inexperienced hands should be avoided. Previously it was the common practice to provide a graduated arc of 5° or 10° at each end of the needle box, but nothing is accomplished by this plan and the size of the accessory is greatly reduced by the present arrangements.

Theodolite Diaphragms, used for geodetic work or field astronomy are touched upon on p. 108. No. 9 is used for a combination of stadia and solar work, but the cut does not show a strict proportion of spacing. The stadia interval is 34′ 23″, while the average diameter of the sun, measured in the celestial sphere, is only a trifle over 32′. The solar square, however, is purposely made smaller in order to facilitate centering by dividing the sun's image into equal segments. Cuts 10, 11 and 12 are used for the observation of time stars.

In geodetic operations the style of signal used depends on the distance, the atmospheric conditions and the resolving power of the objective. If the vertical wires are twin lines, spaced 20″ apart, as shown in Fig. 80, they will intercept an interval of about 6 in. per mile. If signals are built in this proportion, centering can frequently be accomplished more accurately than by trying to bisect with one wire. The same idea is carried out in the filar micrometer eyepiece of the reading microscope considered on p. 49.

Fig. 80

Glass Diaphragms have been used more or less for a great many years on account of their permanency; but in quickly changing temperatures the surfaces will fog from the condensation of what moisture may be inside the tube. The new cryptic focus, described on p. 83, has nearly, if not quite, overcome this tendency and we have used glass diaphragms more recently with success. The lines are cut with a diamond point and do not need a pigment filling. Such lines, now employed for our Compensation Level (see p. 29), when carefully focused are very sharp and distinct. In Germany they have succeeded in photographing the lines on the glass of late years with an emulsion of extremely fine grain and dense texture. It is said that the back side of this microscopic line can be made white to facilitate illumination for night work.

Glass diaphragms, when supplied with our Theodolites and Tachymeters, are not to be feared as previously. With the 4½-inch instrument the diaphragm is readily accessible in the possible event of fogging, in both the erecting and inverting telescope. If the ocular mount in either case is unscrewed, the diaphragm will be found to lie immediately underneath. If the dew is on the ocular side, it may be removed with a clean linen handkerchief. If on the objective side, remove the top and bottom diaphragm screws and tip the reticle over, using the side screws as gimbals. After cleaning, replace and readjust for collimation.

With the 5-inch inverting telescope the process is equally simple, but with the 5-inch erecting telescope the whole eyepiece

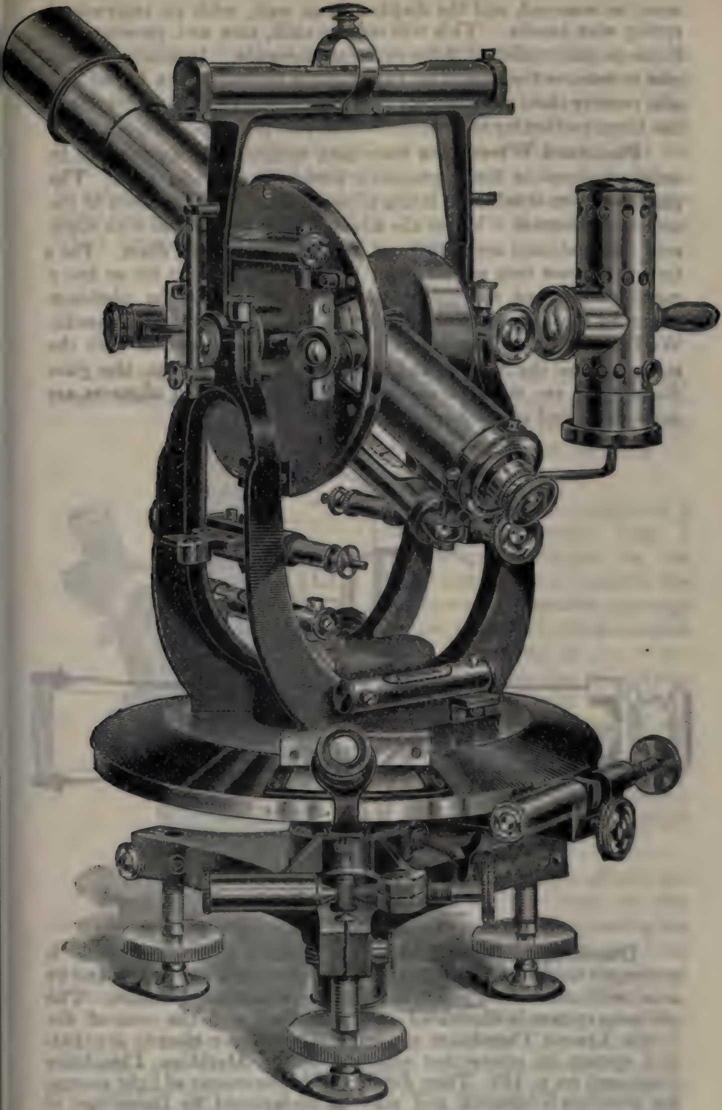

Fig. 81.—No. 706 7-in. Theodolite with European 3-screw Base, Beveled Plate, Axis Illumination, Striding Level, Attached Magnifiers, etc.

must be removed, and the diaphragm as well, with an improvised spring wire handle. This will require skill, care and patience but it can be accomplished if the occasion demands. It would be simpler to remove the objective, rack out the focusing lens barrel and also remove that, then improvise a swab and very carefully clean the inner surface by this means. (See also top of p. 102).

Platinum Wires have been used considerably in the past in order to overcome the hygrometric properties of spider webs. The platinum wire is soldered into a silver tube and drawn down to the uttermost fineness. When the silver coating is eaten off with nitric acid, the platinum core must not exceed .005 mm in diam. For a time they were very popular, but it is not so easy to fix so fine a metallic filiament to another metallic surface and the platinum seems to loose some of its ductility and breaks off under shocks. We frankly confess that we have never had much success with the scheme. For the Nos. 90 and 92 Alidades, for instance, the glass diaphragms are more permanent and even in humid climates are dependable and altogether satisfactory.

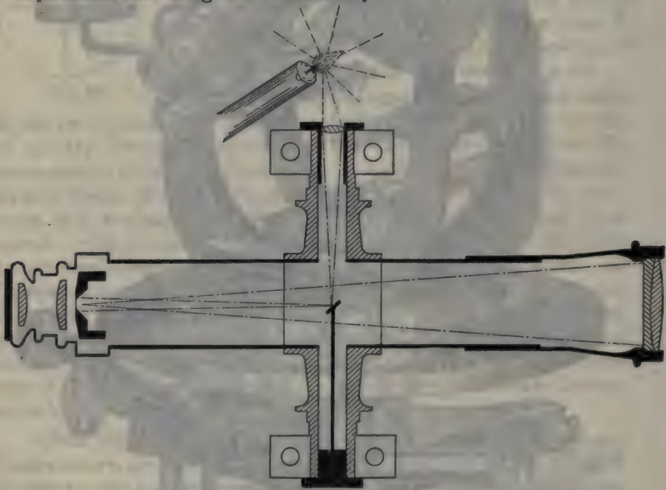

Fig. 82

Diaphragm Illumination for mining and night work is accomplished by the Illuminator Tube described on p. 187, and by axial perforation with either oil-lamp or electric illuminant. The oil-lamp system is illustrated in connection with the cut of the 7-inch Transit Theodolite on page 143 and the electric dry battery system in connection with the 8-inch Meridian Theodolite illustrated on p. 149. They differ only in the matter of light source; the principle is identical and may be understood by inspection of Fig. 82.

The telescope axis is perforated at both sides. Into one side is inserted a small lens mount by which the rays are concentrated upon a very small 45° reflector, as shown in the illustration. The long slender mounting for the reflector is attached to a plug which is inserted into the opposite axis perforation as indicated. If the illumination is so brilliant as to obscure the rest of the field, it can be subdued by slightly turning the plug which sustains the reflector mounting. In this respect it is superior to the polished sphere which has also been used for this purpose.

The Micrometer Microscope

Micrometer microscopes are usually employed in pairs on the larger and more finely divided circles of alt-azimuths in place of the vernier scales commonly used for smaller and less accurate instruments.

Fig. 83

This appliance consists of a compound microscope with a movable twin filament in the focal plane that is intended for the measurement of smaller spacings than are indicated on the graduated circle. According to one of the fundamental theories of microscopy, the objective of the microscope produces an image of the spaces and numbers at the micrometer diaphragm, the size of which depends upon its focal length and its distance from the graduated circle.

The focal length of the objective and the pitch of the micrometer screw are fixed conditions, but by carefully regulating the distance between the objective and the graduations, an image of the spaces can be formed that will exactly equal the pitch of the thread or some multiple thereof. If the limb is divided into $\frac{1}{6}°$ spaces, for instance as shown in Fig. 84, we may utilize the conditions given as above so that it will take five revolutions of the screw to draw the twin wires over one of the image spaces. This makes it very convenient to assign $\frac{1}{5} \times \frac{1}{6}°$, or 2′, to each revolution of the drum, and if the drum is sub-divided into 60 equal spaces we can allow $\frac{1}{60} \times 2'$, or 2″, for each division of the drum. In this case both drum heads are numbered 0, 10, 20, 0, 10, 20. The reason for this is that the

mean of two microscope readings is desired; and since, in this relation, the value of one division is 2″, the mean value sought in seconds is simply the sum of the two drum readings. Other combinations can be employed by which each sub-division on the drum can be made to represent 5″, or 1″, as the case may be.

Adjustments

1. To secure distinct vision of the cross-wires, the eyepiece must be moved out, or in, until the wires are clearly and sharply defined. This adjustment is independent of all others. It differs for different persons and is the first one to be attended to in using the microscope.

Fig. 84

2. To make an even number of turns of the screw equivalent to a given space, measure the image of the space with the screw. If the image is too small, the objective must be brought nearer to the graduation and the cross wires moved further from the objective; opposite motions of the parts must be made if the image is too large. The tubes carrying the objective and micrometer box permit such motion. A few trials will make this adjustment sufficiently close. In making this adjustment care must be taken to avoid parallax which occurs when the cross wires and the image of the object viewed are not in the same plane. It is detected by moving the eye from side to side while looking through the ocular. If the twin wires and image show any relative motion parallax exists. It may be removed (supposing the ocular adjustment to be made) by moving the whole microscope nearer to, or further from, the graduations.

3. To bring the zero of the comb scale into coincidence with the cross wires when the micrometer drum reads zero, move the comb scale by means of the adjusting screw at the end of the micrometer box. This adjustment need not be very exact since the only office of the scale is to count whole revolutions. It may be also accomplished by moving the micrometer head on the screw shaft since the head is usually held fast by means of a lock nut on the shaft.

4. To place the two opposite microscopes 180° apart, set one of them at zero and bring a graduation line to bisect the wire interval. Then the other microscope may be brought to bisection on the opposite line by moving the drum on the screw shaft and by adjusting the comb scale to suit, it may be made to read within a few divisions of the first microscope. Close agreement is not essential, but it is convenient to have both microscopes read the same to the nearest minute.

Method of Reading

This will be best understood by considering a special case. Suppose it is required to read the two opposite micrometer microscopes of a theodolite whose circle is divided into 10-minute spaces. Let 5 revolutions of the screw be equivalent to one of these spaces; then one revolution is equivalent to two minutes and the micrometer drums will be assumed to be divided into 60 equal parts and numbered from zero to thirty, twice. The relations to be considered are illustrated in the inserted diagram which shows a degree of a circle, the position of the principal points of the microscope objective, the position of the micrometer threads, *t*, *t*, etc. In this diagram the line *bc*, *b'c'*, is the line defined by the micrometer wires (or the point midway between them) when the micrometer reads zero revolutions and zero divisions. This line falls between the 40′ and 50′ lines of the circle. The reading of the circle is 17° 40′ plus the distance *ab* expressed in angular measure. The image-equivalent of *ab* is *a'b'*, and this is measured by moving the micrometer wires until the space between them is bisected by the image of the 40′ line *a* or by *a'*. Suppose the distance *a'b'* is three revolutions (counted by three depressions of the comb scale) and 8.3 sub-divisions of the head. Then the complete reading is 17° 46′ 16″.6

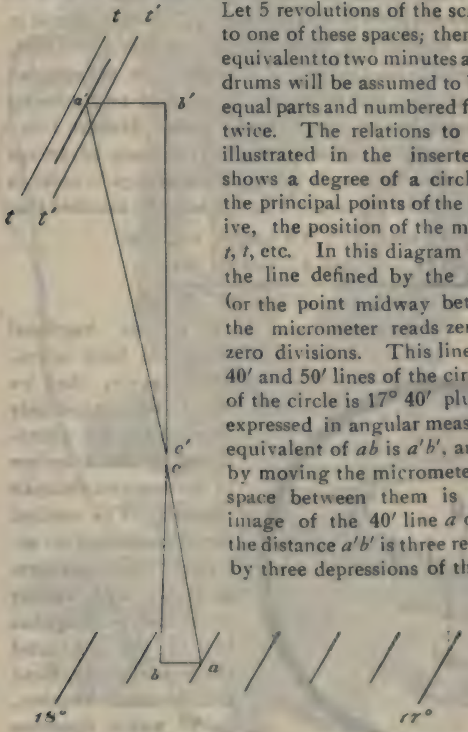

Fig. 85

If the opposite micrometer reads 197° 46′ and 11.9 divisions, the mean reading of the circle is (using the degrees indicated in the first microscope), 17° 46′ 20″.2 since $\frac{1}{2}(8.3 + 11.9) \times 2'' = 20''.2$.

It should be observed that the micrometer drum readings increase as the screw is turned backward, but in bringing the twin wires to bisection on any division line *the screw should always be turned positively* so as to pull the diaphragm against the springs which hold the micrometer screw in its bearings.

All vernier contact edges whether on flat, beveled, or edge graduated circles are exposed to more or less attrition through the accumulation of dust and constant use. In Europe the vernier scales are being replaced to some extent by a modified form of reading microscope which has no micrometer drum, but a finely ruled glass diaphragm in the common focal plane, by which the smallest sub-division of the limb can be still further sub-divided.

Thus in Fig. 86 the limb is divided into $\frac{1}{3}°$, which is not at all unusual for 5-in. instruments, and each degree line numbered. A space equal to 20′, used at the diaphragm, is sub-divided into 10 equal microscopic spaces having a value of two minutes each. The long index line is the zero of the scale. If these index lines are not diametrically opposite in the microscopes, advantage of this fact may be taken by simply adding the readings in order to get the true result. Referring to the diagram let it be assumed that

Micro I reads 69°, 3.6 spaces, and that
Micro II indicated 3.7 spaces, then

Mean = 69°, 7′.3 = 69° 7′ 18″

If the fractional values in both microscopes agree, then we should be required only to multiply the micro-spaces by two to reduce to minutes and decimals thereof. The method is recommended for accuracy that compares favorably with vernier scales and prolongs the life of the graduated circle indefinitely. Read also Decimal Verniers, p. 99 which dispenses with calculations involving minutes or seconds.

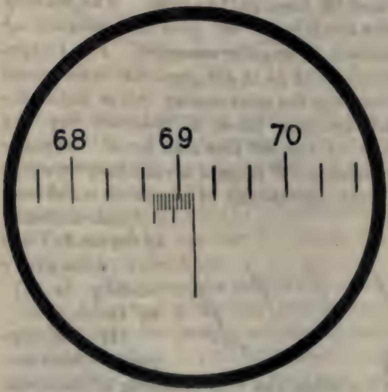

Fig. 86

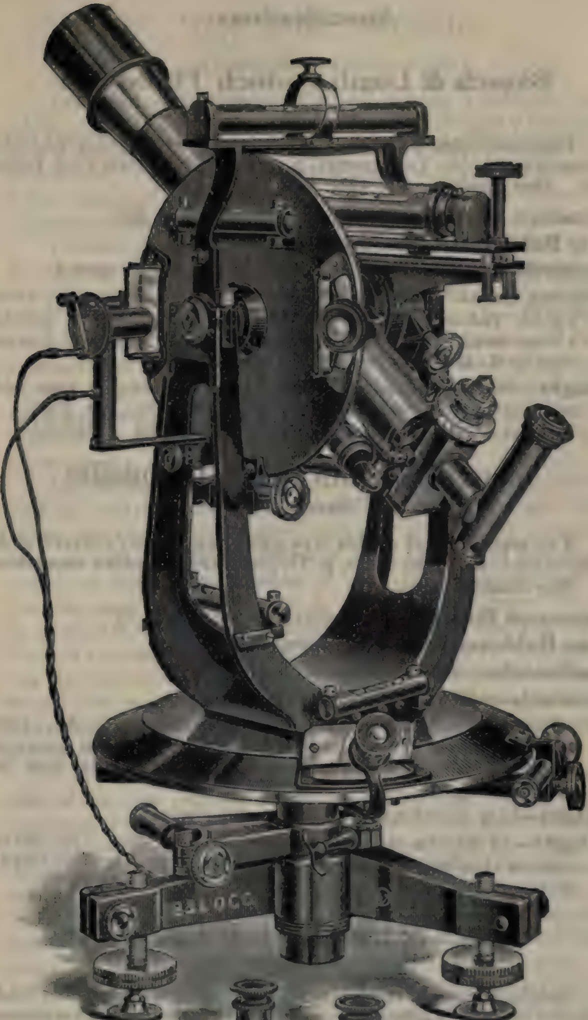

Fig. 87—No. 806 8-in. Theodolite with European 3-screw Base, Beveled Plate, Electric Axial Illumination, Latitude Level, Striding Level, Micrometer Diaphragm, Diagonal Eyepiece, etc.

Bausch & Lomb 4½-inch Theodolite

"Orographic"

The specifications on this type are quite generally covered under those of the 4½-in. Tachymeter, p. 126, with the following exceptions: (See illustration p. 128).

Telescope Bubble—20″ sensitiveness instead of 30″.

Plate Bubbles—2½ in. long instead of 1½ in.

Standards—Aluminun bronze, 5½ in. high, 2½-in. spread.

Leveling Base—When furnished with the European model, arms, 2¼-in. radius; 3¾ in. between centers. Leveling screws, 1¼ in. head. May be furnished also with ordinary 4-screw base or B. & L. 3-screw base; (see p. 139).

Weight—With European leveling base, 9¾ lbs.; in case with accessories 18 lbs. Tripod No. 74-T, 9½ lbs., 5 ft. high. Heavier tripod with Reducing Ring furnished if desired.

Bausch & Lomb 5-inch Theodolite

"Stadiametric"

The specifications on this type are quite generally covered under those of the 5-in. Tachymeter, p. 127, with the following exceptions: (See illustration, p. 132).

Telescope Bubble—20″ sensitiveness instead of 25″.

Plate Bubbles—2½ in. long instead of 2 in.

Declinatoire—4-in. tubular needle.

Standards—6⅝ in. high, 3-in. spread

Leveling Base—When furnished with European model, arms, 2½-in. radius; 4⅜ in. between centers. May be furnished also with ordinary 4-screw base or B. & L. 3-screw base (see p. 139).

Height—12¾ in. with arc; 14¾ in. with circle.

Weight—14 lbs.; in case with accessories, 25 lbs. The heavier tripods supplied with the 6-in. instrument will be furnished if so requested.

Bausch & Lomb 6-inch Theodolite

"Tunnel"

Specifications on this type are quite generally covered under those for the 6-in. Tachymeter, p. 127, with the following exceptions: (See illustration, p. 137).

Plate Bubbles—2½ in. long instead of 2 in.

Height—13 in. with arc; 14¾ in. with circle.

Weight—17 lbs; in case with accessories, 30 lbs.

The Solar Meridian

Direct Solar Observation

Inasmuch as the direct method of deriving a solar meridian is widely exploited, we feel it a privilege to undertake a fresh review of the argument. Unless the theodolite is equipped with a solar attachment, this is the only reliable means available for establishing a true meridian in the daytime with the single exception of the Canadian method of observing polaris.

Compared with the solar attachment it is contended* that the theory is not more complex; that while the computations are longer they are not beyond the ability of the modern surveyor; that no other instrumental equipment is necessary than a good transit with accurate vertical circle and perhaps an eyepiece prism or diagonal eyepiece with a sun filter of dense glass; that check observations can be made in more rapid succession; that no preliminary preparation or computations are necessary, and that by the system of reversals required good results may be expected when the transit is out of adjustment.

On the other hand, the moment of observation is recorded more precisely and the extensive computation entailed by check observations offers repeated opportunities for error in the calculations. Observations should not be made when the sun is less than 15° above the horizon on account of uncertain refraction, nor at any time between 11 a. m. and 1 p. m. when the movement of the sun in altitude is insignificant compared to his movement in azimuth. Double reversed observations do not compensate for errors in leveling the instrument. Any appreciable error in this respect (see I-b, p. 63) will cause discrepancies in the measured altitude and in the final bearing. If there is an index error in the vertical circle, it should be noted. (See VII, p. 71.)

The difficulty of bisection with ordinary cross wires, due to the size of the sun, requires attention and special expedients. The diameter of the sun is 32′, or thereabouts, in the vertical plane, but his diameter in the horizontal plane varies as $32' \div \cos \text{Alt}$. On this account some operators take the average of several bisections. An expert observer will be able to make a double set of observations including eight pointings on the sun, four on the reference mark and eight watch readings within 15 to 20 min. One set of observations will be taken with the telescope erect and the second with it inverted. This implies the necessity of having the transit equipped with full vertical circle. The average of six readings, or even four, ought to give reliable data for the sun's center, but one should not be excused for thus attempting to correct the errors of a poorly adjusted instrument.**

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* *See Prof. W. H. Burger in Trans. Ill. Soc. Surv. & Eng., 1912.*

** *See Prof. C. E. Rowe in Mines & Minerals, Mar. 1910, p. 488.*

The best time for observation is in mid-morning and in mid-afternoon when the errors of refraction are comparatively small and when the sun is changing rapidly enough in both altitude and azimuth to insure the best results.

It is necessary to know the latitude of the place to the nearest minute. This is found by measuring the altitude of the sun at apparent noon, subtracting the refraction and adding parallax correction. From this, the zenith distance can be computed by subtracting the result from 90°. The latitude can then be determined by the algebraic sum of the zenith distance and the declination. (Refer to p. 168.)

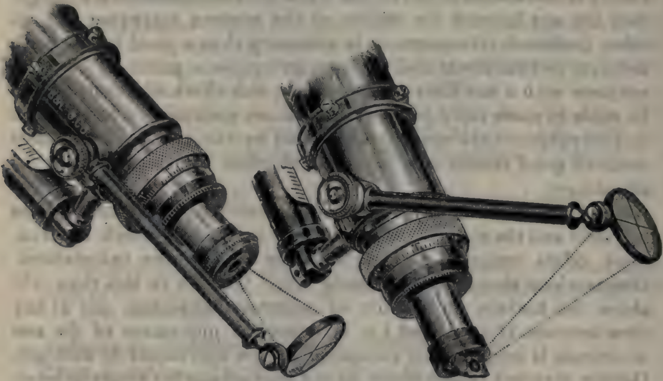


Fig. 88

The appliances that have been contrived to facilitate direct observation on the sun include the eyepiece cap with combination shutter and ray filter as shown in Fig. 89, the eyepiece prism for the steeper sights, with the swiveled moderation glass as shown above, and discussed on p. 191 and the Davis Solar Screen, (Fig. 88) designed by J. B. Davis, erstwhile Prof. Civ. Eng. at the Univ. of Mich. The eye cap with ray filter is furnished with our Tachymeters and Theodolites without extra charge when specified; but the Davis Screen is made only to order because many engineers prefer to use a white card, or the page of a notebook, to catch the projected image of the sun and cross wires.

Fig. 89

For this purpose the eyepiece is to be drawn out somewhat, and whether the Davis Screen or card is used a position can be found by experiment where the image can be sharply focused.

It would seem that the card system would offer some inconvenience in trying to follow the sun with two tangent screws and hold the card at the same time but it has been customary to clamp the instrument when the telescope has been approximately set, then by setting either wire a little ahead of the sun's apparent motion, keep the other wire tangent to his image and allow the motion of the sun to arrive at the other position of tangency.

Theory

Referring to Fig. 90*, O is the position of the observer, Z is the zenith directly above him, P is the celestial pole at an altitude above

Fig. 90

the north horizon equal to the latitude of the place of observation, and S, the sun as it would appear in the morning in north declination. The line SO should properly be drawn to the center of the

* *Louis Ross, U. S. Deputy Surv., 208 Market Street, San Francisco, Cal., in Eng. News, Mar. 6, 1913. For Mr. Ross' original observations on this subject, see Harvard Eng. Jour., Jan. 1910.*

earth and would pass through the observer were he anywhere between the topics at a latitude equal to the sun's declination. Otherwise there is a slight error of parallax increasing from 0″ at the equator to less than 9″ at the pole, so that it is usually neglected. Having found the altitude, **h**, the declination, **d**, and the latitude, **l**, the complements of these three angular values constitute the spherical triangle PZS, which can be solved by the formula:

$$\cos \tfrac{1}{2}\,\mathrm{Az} = \sqrt{\frac{\sin \tfrac{1}{2}s \times \sin(\tfrac{1}{2}s - \text{co-dec})}{\sin \text{co-alt} \times \sin \text{co-lat}}}$$

in which s = sum of 90° — **h**, 90° — **d**, and 90° — **l**. Also we have,

$$\sin \tfrac{1}{2}\,\mathrm{Az.} = \sqrt{\frac{(\sin \tfrac{1}{2}s - \text{co-lat})\,(\sin \tfrac{1}{2}s - \text{co alt})}{\sin \text{co-alt} \times \sin \text{co-lat}}}$$

Mr. Ross developed and used the following simplified formula:

$$-\cos \mathrm{Az.} = \frac{(\sin \mathbf{h} \times \sin \mathbf{l}) - \sin \mathbf{d}}{\cos \mathbf{h} \times \cos \mathbf{l}}$$

In Johnson-Smtth, p. 99, attention is directed to the fact that in spherical trigonometry,
$\cos(90° - \mathbf{d}) = [\cos(90° - \mathbf{h}) \times \cos(90° - \mathbf{l})] + [\sin(90° - \mathbf{h}) \times \sin(90° - \mathbf{l})] \cos \mathrm{Az.}$ from whence another simplified formula:

$$\cos \mathrm{Az.} = \frac{\sin \mathbf{d}}{\cos \mathbf{h} \times \cos \mathbf{l}} - (\tan \mathbf{h} \times \tan \mathbf{l})$$

The sign of the first term in the second half of the equation will be minus if the declination is south, and the second term will be plus if the latitude is south. If the *cos* Az. is plus, the azimuth is between 0° and 90° as measured from the north; if minus, it is between 90° and 180°. For observations in the southern hemisphere, use minus signs for north declination and refer azimuth to the south pole.

The application of the theory can be understood by following a specific case, using the second formula given above. Let it be supposed that we are measuring azimuth to a line that was thought to be a true meridian. The example suggests the practice of observing the sun in the diagonally opposite corners of the diaphragm on the assumption that it is easier to point to the edge than the center of the sun's disc; but Mr, Ross reports that, in actual practice, the center of the sun can be determined by an average of four or six pointings within a probable error of 1′.

Example

1.	⊕	9h, 39m, 50s.	141° 02′ 00″ = Az.;	33° 34′ 00″ = h.
2.	⊕	9h, 43m, 50s.	141° 26′ 00″ = Az.;	33° 31′ 00″ = h.
3.	☉	9h, 41m, 50s.	141° 14′ 00″ = Az.;	33° 32′ 30″ = h.

Decl. at Greenwich mean noon, March 3rd., 19—	7° 02′ 00″
Correction, 57″.37 × 3.7 - - - - - -	03′ 32″
True Declination - - - - - - - -	6° 58′ 28″
Co-declination, (90° — d) = SP - - - -	96° 58′ 28″
Apparent Altitude - - - - - - -	33° 32′ 30″
Refraction Correction for 2⅓ hours, (see page 164)	01′ 16″
True Altitude - - - - - - - -	33° 31′ 14″
Co-Altitude, (90° — h) = SZ - - - -	56° 28′ 46″
Let it be assumed that the latitude is - - -	41° 28′ 48″
Then the co-latitude, (90° — l) = ZP - - -	48° 31′ 12″

Co-dec = 96° 58′ 28″

Co-alt = 56° 28′ 46″ — ac *Log sin* = 0.0789966

Co-lat = 48° 31′ 12″ — ac *Log sin* = 0.1254098

s = 201° 58′ 26″

½ s = 100° 59′ 13″

½ s—SZ = 44° 30′ 27″ — *Log sin* = 9.8457196

½ s—ZP = 52° 28′ 01″ — *Log sin* = 9.8992743

2) 19.9494003

Log sin ½ Az. = 9.9747001 = 70° 37′ 56″

True Az. = 141° 15′ 52″

Measured Az. = 141° 14′ 00″

Error of Line = 01′ 52″

The Ross Meridiograph *

The expert investigations of Mr. Ross into the mathematics of direct meridian determination has developed a circular slide rule of logarithmic scales by which the sun's azimuth may be rapidly calculated in the field.

The declination, the latitude and the sun's altitude are to be established by any of the well known processes. These values are laid off on the proper scales, and the azimuth is to be read off directly by inspection.

It consists of two circular celluloid discs and a reading arm rotating about a common center. On the discs are graduated eight separate logarithmic scales of altitude, latitude, declination and

* *Consult also Eng. News, Feb 16, 1914; Colliery Engineer, June 1914, Eng. & Cont. July 15, 1914, etc.*

bearing to the nearest 5′ or 10′. Each scale is designated at its proper radial distance on the reading arm. To use the appliance:

First, the altitude is set against the latitude on scale **a**. Opposite the index, read number A.

Then, the altitude is set against the latitude on scale **b**. Opposite the given declination, read number B.

Finally, Opposite the number A + B read the true bearing of the sun.

The scale A is arranged to solve *tan* Alt. × *tan* Lat.

The scale B is arranged to solve *sec* Alt. × *sec* Lat. × *sin* Dec.

A + B = − *cos* bearing.

This arrangement evidently solves the elemental cosine law of spherical trigonometry involved in the third formula on p. 154

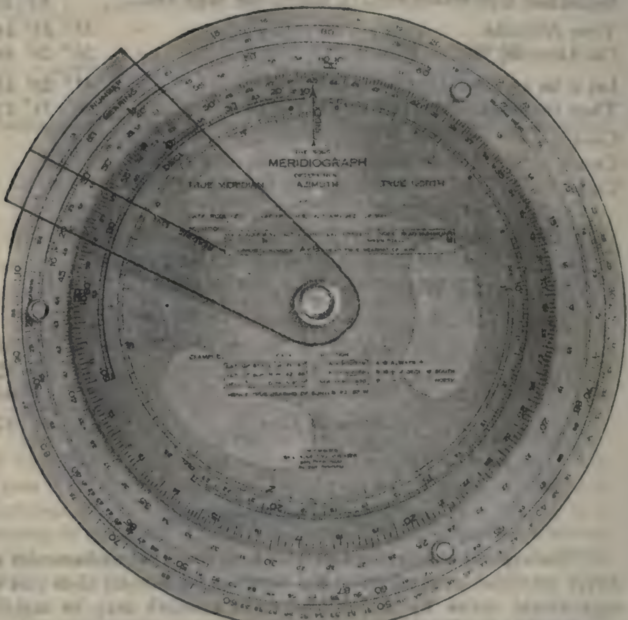


Fig. 91

The basic advantage of this appliance lies in the magnification and legibility of the scales. On the outer 7-inch circle, the spaces are as open as one might expect to find the divisions, on a protractor 5 feet across. It is claimed that the accuracy of the Meridiograph keeps pace with that of observation. A reading can be taken to the nearest minute on the assumption that the user will be able to interpolate a space of 1/200 inch. The sufficiency of this allowance has been checked on numerous sets of data.

THE SOLAR ATTACHMENT

IN ORDINARY compass surveys, the elimination of the effect of local attraction is an elementary consideration, but in the azimuth method an arbitrary assumption that the first course is free of such influence must govern the entire calculation; so that we have an undetermined error applied to every deduction unless the *True Meridian* is previously established at the first station by means of a solar attachment, of which the Saegmuller type is the best known.*

Unless such a meridian is first established, the sum of the interior angles is no proof of the accuracy of the work, even though it should equal the proper theoretic aggregate. Empirical formulæ for the declination of the needle at any given place are not always reliable, and it is better that the needle's declination should be determined by comparison with the Solar Meridian, which is readily and accurately obtained as hereinafter set forth.

The Solar Attachment is a telescopic device mounted equatorially at the top of the Surveyor's Transit for the more accurate determination of the meridian by an observation on the sun. The cut, p. 158, shows the latest improved model with a 12-power, interior focusing telescope mounted in standards that are a part of a continuous metallic frame. The arch above is of sufficient height to permit inclinations of the solar telescope of somewhat more than 23° 30′ to the axis of the main telescope.

The solar telescope is mounted so as to revolve in azimuth about its "polar axis", which is adjusted to perpendicularity with the sight line of the main telescope as well as with the horizontal axis of the transit, as hereinafter described.

At the top of the telescope tube is mounted a sensitive level vial which dispenses with the necessity for a specially contrived declination arc. The two target sights mounted at the ends of the bubble tube are used to direct the solar telescope toward the sun. When the shadow of one is cast upon the other, the sun is in the field of view. The "hour circle" furnished with the earlier models of this attachment is therefore another needless elaboration.

**See Surveying Manual, Pence & Ketchum, 3rd. Edn., 1909, p. 127, also Amer. Civ. Eng. Pocket Book 1st Ed., 1911, p. 24.*

This attachment can be used when the sun is partly obscured by clouds, when the lens-bar of the Burt model fails altogether. The telescopic sight also has a great advantage in favor of accurate centering. The sun's amplitude in the celestial sphere is equal to about 32′. An attempt to center an image of this angular value between the cross lines on a lens-bar offers unusual opportunities for error in declination.†

Fig. 92—Equatorial Solar Attachment with Swivel Adapter Base
Pat. by G. N. Saegmuller, May 3, 1881; Apr. 13, 1909.

The following table, prepared by the late Prof. J. B. Johnson of Washington University, St. Louis, Mo., will show the effect of such errors. He says:-

"This table is valuable in indicating the errors to which the work is liable at different hours of the day and for different latitudes; as well as serving to correct the observed bearings of lines when it afterwards appears that a wrong latitude or declination has been used. Thus, on the first day's observations I used a latitude in the forenoon of 38° 37′, but when I came to make the meridian observation for latitude I found the instrument gave 38° 39′. This was the latitude that should have been used, so I corrected the morning's observations for two minutes error in latitude by this table."

† *See Engineer's Surveying Instruments, I. O. Baker, 2nd. Edn, 1906, p. 62.*

Errors in Azimuth (by Solar Observation) for 1 Minute Errors in Declination and Latitude.

Hour	For 1 Min. Error in Declination				For 1 Min. Error in Latitude			
	Lat. 30°	Lat. 40°	Lat. 50°	Lat. 60°	Lat. 30°	Lat. 40°	Lat. 50°	Lat. 60°
	Min.	*Min.*	*Min.*	*Min.*	*Min.*	*Min.*	*Min.*	*Min.*
11.30 A. M. / 12.30 P. M.	8.85	10.00	11.92	14.07	8.87	9.92	11.82	13.56
11.00 A. M. / 1.00 P. M.	4.46	5.04	6.01	7.68	4.31	4.87	5.81	6.37
10.00 A. M. / 2.00 P. M.	2.31	2.61	3.11	4.00	2.00	2.26	2.69	3.46
9.00 A. M. / 3.00 P. M.	1.63	1.85	2.20	2.83	1.15	1.31	1.56	2.00
8.00 A. M. / 4.00 P. M.	1.33	1.51	1.80	2.31	0.67	0.75	0.90	1.15
7.00 A. M. / 5.00 P. M.	1.20	1.35	1.61	2.07	0.31	0.35	0.42	0.54
6.00 A. M. / 6.00 P. M.	1.15	1.31	1.56	2.00	0.00	0.00	0.00	0.00

Note—Azimuth observed with erroneous declination or co-latitude may be corrected by means of this table by observing that for the line of collimation set *too high* the azimuth of any line *from the south point* in the direction S. W. N. E. is found *too small* in the *forenoon* and *too large* in the *afternoon* by the tabular amounts for each minute of error in the altitude of the line of sight. The reverse is true for the line set too low.

Theory

In his path along the ecliptic, the sun occupies an angular position with respect to the equator and other elements in the celestial sphere which can be very accurately calculated in advance, and for which tabular values, called the Solar Ephemeris, can be prepared years ahead of time.

These conditions being known, it is not difficult, by use of the graduated circle and spirit levels, to set the main telescope parallel to the plane of the earth's equator and to establish a relationship between the two telescopes that shall be equal to sun's declination for any particular hour of any particular day. The solar telescope, revolving on an axis that has been set parallel to the earth's poles, will not, and cannot, follow the sun in his diurnal path unless the main telescope is in the plane of the meridian.

This process constitutes an accurate mechanical solution of the spherical triangle for the determination of the direction of the line ZP, Fig. 93, or the projection of that line on the surface of the earth.

The accuracy of solar observations depends primarily upon the adjustment of the transit and solar attachment, the latitude allowance, the skill of the operator, the accuracy of his computations and atmospheric conditions; but employing proper values, which are not in the least difficult of determination, the true meridian can be obtained within a minute of arc or nearer, and within 10 minutes of time.

Merrimam considers the determination of the true meridian the most difficult undertaking in topographical surveying. While we therefore go into the theory at some length, we have never considered the routine either ponderous or complicated. As Professor Tracy would say, there is something to remember but not much to understand. Before taking up adjustments and use of the attachment, the theory of its application and the physical concepts involved should be fixed in the mind by reference to the inserted diagram.

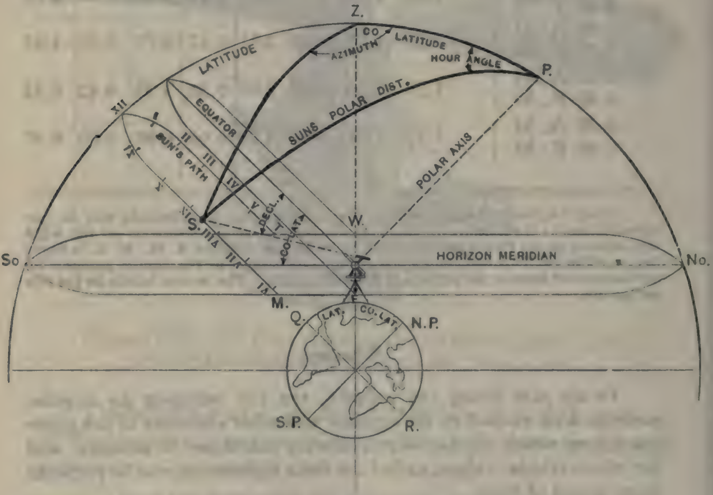


Fig. 93

The Celestial Sphere, or hemisphere, is that portion of the heavens which is visible to the observer at any point on the earth's surface. For the sake of this argument its radius, while infinite, may be assumed as reaching from the center of the earth to the sun.

In Fig. 93 an exaggerated state of affairs is shown in which So. ZPNo. is the plane of the observer's meridian and the line So. No. is the true meridian lying in the observer's horizon and passing through the center of the instrument. An observer stationed at the instrument can see only those terrestrial objects within the range of his vision, but taking the sun at 93 million miles, or a distance nearly

12,000 times the diameter of the earth, creates the effect of reducing the world to a mere celestial speck, changing the field of view virtually from its surface to its center.

Parallax is the difference between the apparent altitude of a celestial body as observed from the earth's surface or from its center. In solar observations it is equal to the angular value of the earth's radius as viewed from the sun, or $8''.94$ in the horizon and nothing in the zenith. In solar observations for the meridian, it is a negligible quantity, but if a correction were to be applied it would always be plus where refraction is minus. It may be found by multiplying the solar parallax, as given above, by the cosine of the observed altitude.

The Zenith is in the plane of the observer's meridian at the point Z, marking a position where the direction of a plumb line, prolonged upward, would pierce the celestial sphere.

The North Pole, P, is as far removed from the zenith as the earth appears to be inclined on its axis. The sun, in his apparent diurnal path, rising at M and setting at T, will, at each moment in the day, occupy a certain relationship with respect to the fixed points, Z and P. It follows then, that the spherical triangle, ZPS, is one that is constantly changing in proportions, depending upon the sun's position above or below the equator and the hour angle, or his position in his daily path. In the morning the angle ZSP is constantly diminishing until at noon, XII, the spherical triangle has become a straight line. Between XI and I the angle ZSP is too small to be accurately solved by this process. On the other hand, from sunrise up to about 8 o'clock, atmospheric refraction makes the calculations rather uncertain. Between 8 and 11 a. m. and 1 and 4 p. m. is the best time of day for solar observation.

The Sun's Declination is his angular distance above or below the celestial equator as measured from the center of the earth. It is regarded as positive when north and negative when south. The declination is 0° at the vernal and autumnal equinox on March 21st and September 23rd when the sun crosses the equator. The hourly change in declination is nearly a minute of arc at these dates. On June 22nd and December 22nd in the summer and winter solstice, the declination has increased to approximately 23° 27′ when the hourly change is nearly zero.

The Sun's Polar Distance is equal to 90° plus or minus the apparent declination, depending upon whether south or north of the equator. This is the angle the solar telescope makes with the prolongation of the polar axis, when pointed at the sun.

The Latitude is the angular distance of any point on the earth's surface, like E, from the equator, Q; or it may be otherwise expressed as the declination of the zenith. The angular distance of the zenith from the pole is therefore equal to the co-latitude of the place of observation, or 90°—L. The degrees of latitude run parallel with the equator at varying distances, averaging 69 miles apart, depending upon the rotundity or oblateness of the zones traversed.

It is assumed that the transit is in perfect adjustment, especially the telescope level and the vernier of the vertical circle. If there is any index error in the vertical circle, it should be carefully rectified, or an allowance should be made when laying off the declination and the co-latitude. The collimation should be perfect for all distances and the telescope axis should be truly horizontal.

1. *The Polar Axis must be at right angles to the plane defined by the line of collimation and the horizontal axis of the main telescope.*

Level the transit carefully so that the telescope bubble will remain centered during an entire revolution. Turn the solar bubble over either pair of adjusting studs which operate at the base of the attachment. Center bubble with tangent screw and revolve the attachment on the polar axis a half turn.

If the bubble runs off center, correct half the error with the adjusting studs and the other half with the little clamp-and-tangent movement of the solar telescope. Revolve the attachment a quarter turn so as to make the test over the other set of adjusting studs. If the bubble shows error, correct all of it in the studs, but repeat the operation over both sets of leveling studs until the solar bubble remains centered during an entire revolution.

In this adjustment the solar telescope bubble has been utilized as a matter of convenience, and for this purpose its relationship to the line of sight in the solar telescope does not enter into the consideration. Incidentally, however, the solar telescope bubble has been adjusted to right angles with the polar axis and in this position is parallel with the sight line of the main telescope.

The little Check Bubble, mounted on the base of the solar standard, should now be adjusted to the center of its run and should remain centered during a revolution, either on the polar axis or on the vertical axis of the transit. Therefore, the Check Bubble should be used for the purpose of rough preliminary setting of the polar axis when the solar attachment is first applied.

2. *To Adjust the Vertical Wire.*

Sight on a plumb line, and if the vertical wire cuts across it diagonally, remove ferrule from diaphragm mount and setting the screw driver blade against one of the washers, tap gently in the direction desired until a coincidence with the plumb line is effected.

3. *To Adjust Horizontal Axis of Solar Telescope.*

While the diaphragm cross is still centered on the plumb line and the adjusting studs have been secured, open the clamp of the solar telescope and, revolving it up and down over limitations defined by the arch of the standard, note if the intersection of the cross wires passes from one side of the plumb line to the other.

If so, loosen the jamb-nut at the right side of the solar telescope axis and turn the capstan-head stud with an adjusting pin until the desired result is secured. When complete, re-set the jamb-nut and test again. No. 2 and 3 adjustments are inter-related and should be conducted together.

4. *The Horizontal Wire must be adjusted in height so that the sight line will be parallel with that of the main telescope when both telescopes bubbles are centered; or angles laid off on the vertical circle of the transit, will not communicate equivalent inclinations in the solar telescope, as referred to the horizon.*

Measure the distance between the centers of the two telescopes by any convenient means. Tack up a sheet of white paper, preferably at one to two hundred feet from the instrument. The main telescope being in a horizontal position, mark a point on the paper as indicated by its horizontal wire. Measure upward an amount equal to the eccentricity of the solar telescope and, with its bubble carefully centered, check the central horizontal wire against the higher point. If it does not coincide, correct the entire error by moving the solar diaphragm up or down the required amount.

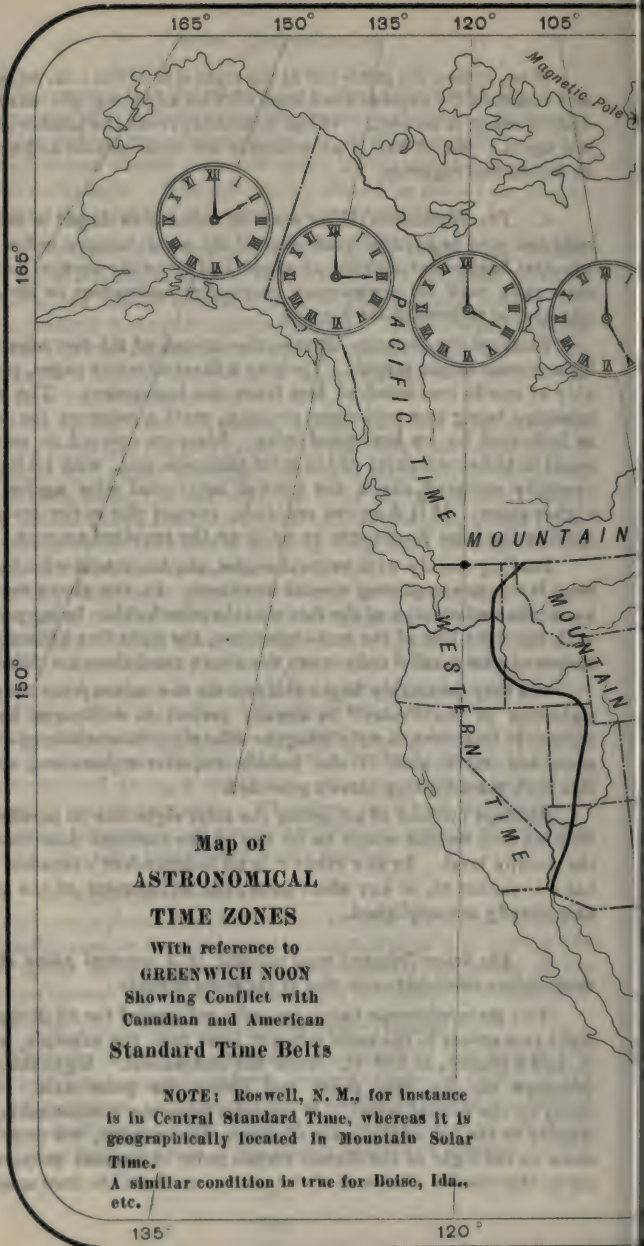
Dealing mainly with vertical angles, the horizontal wire in the solar is the one requiring special attention. In the above test we have taken advantage of the fact that the solar bubble, being parallel with the sight-line of the main telescope, the sight line of both telescopes can be parallel only when the above conditions are fulfilled.

We may preferably begin this test on the assumption that the sight-line of the "solar" is already perfect as collimated by the maker. In this event, simply bring the solar sight-line to bear upon the upper test mark and, if the bubble requires adjustment, secure this with the adjusting screws provided.

Another method of adjusting the solar sight-line to parallelism with its own bubble would be by any of the methods described for the dumpy level. In any event it is an independent operation and has no relation to, or any effect upon, the adjustment of the polar axis already accomplished.

5. *The Solar Telescope must revolve in a vertical plane that is everywhere coincident with that of the main telescope.*

The main telescope being properly collimated for all distances, sight two points in the same straight line: one, for example, at 50 ft. and a second, at 150 ft. from the instrument. Sight the solar telescope on the near point and clamp the polar axis. Now focus on the distant point. If the solar telescope is mounted eccentrically to the left of the main telescope, for instance, the error will show to the right of the distant transit point in amount proportionate to the relative distances of the test points from the instrument.



Map of

ASTRONOMICAL

TIME ZONES

With reference to

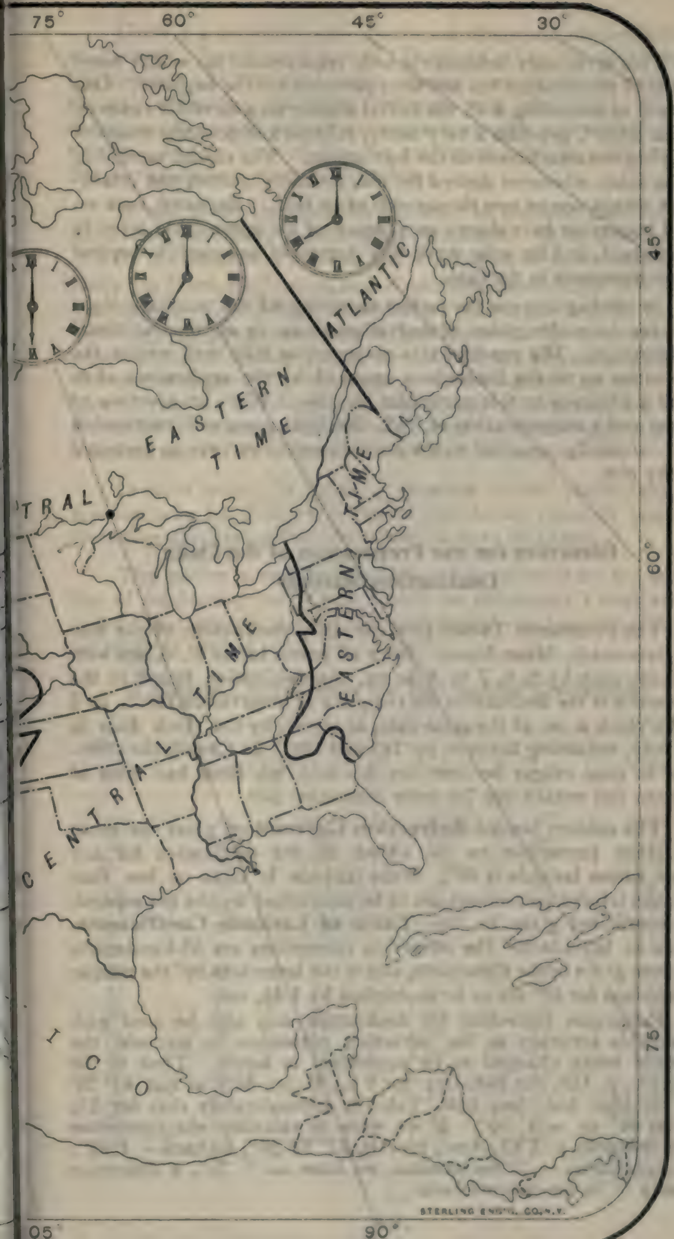
GREENWICH NOON

Showing Conflict with

Canadian and American

Standard Time Belts

NOTE: Roswell, N. M., for instance is in Central Standard Time, whereas it is geographically located in Mountain Solar Time.
A similar condition is true for Boise, Ida., etc.


75°
60°
45°
30°
EASTERN TIME
ATLANTIC TIME
CENTRAL TIME
EASTERN TIME
45°
60°
75°
90°
STERLING ENG'G. CO., N.Y.

This is the only test and the only requirement for which some means of rectification has not been provided for the engineer. Our method of mounting with the swivel adapter on a carefully centered "solar table", provides a very ready, reliable and accurate means of applying the attachment to the instrument. The casual mounting of the solar, whenever desired for use, probably centers the attachment within one or two thousandth of an inch. Repeated tests on rigid inspection have shown an absence of an appreciable error in this respect, and for solar work this degree of accuracy is beyond all requirements in the case.

In mining surveys the perfect centering of the auxiliary sight line, for the prolongation of shaft alignments, is an unconditional desideratum. We consider this construction falls well within the necessities up to the limitations imposed by the application of so small a telescope to this particular purpose. With an aperture of 19mm and a magnification of ×12, the light grasp and penetration are confessedly unsuited to the more extensive surveys in deep and smoky pits.

Direction for the Preparation of the Daily Declination Settings

The Ephemeris Tables give the daily declination of the Sun for Greenwich, Mean Noon. Since all points in the U. S. are west of Greenwich by 5, 6, 7 or 8 hours, the declination found in the ephemeris is the declination for the place of observation for 7, 6, 5 or 4 o'clock a. m. of the same date, as shown by the clock dials in the map, occurring between pp. 163 and 164. In Canada the difference in time ranges between the 4th and 9th hour but most of Mexico lies within the 7th hour correction belt.

The column headed **Refraction Correction** gives the mean refraction correction to be added to the declination for any point whose latitude is 40°. If the latitude is more or less than 40°, the tabular corrections are to be multiplied by the corresponding coefficient given in the **Table of Latitude Coefficients.** Thus in latitude 30° the refraction corrections are 65-hundredths of those given in the Ephemeris, and if the latitude is 50° the tabular corrections for 40° are to be multiplied by 1.43, etc.

Refraction correction for declination may also be used with reasonable accuracy as the refraction correction in altitude, the altitude being changed to its equivalent in hours. Thus in the example, p. 155, the Ref. Cor. for 9 h., 41 m., 50 s. at Lat. 41° 28′ 48″ for Mar. 3rd., (see Eph. Tab.) is approximately that for 2⅓ hrs. at 40° or — 1′ 16″. If we were to calculate the correction from the table, p. XXI, Apx., for Alt. 33° 32′ 30″, we have — 1′ 28″. Correct this by + 7″ for parallax, we have — 1′ 21″, a difference of only 5″ in the two methods.

There is a negligible error in the use of the latitude coefficients as directed. It will never exceed 15″ except when the sun is near the horizon. At that time, solar observations should be avoided, for any refraction correction becomes very uncertain. With a rising barometer and denser atmosphere or with a depressed thermometer, at this time of day the correction may be increased to double the amount given in the tables.

It was probably Claudius Ptolemy in the first century who discovered the effects of atmospheric refraction. It is always positive for the purpose of this calculation, that is, it increases numerically the North declination and diminishes numerically the South declination. This rule is the reverse of the one given under Precise Leveling, p.36; but the conditions are opposed. Having made the observations in leveling we are arriving at the correct position of the object sighted by subtracting the refraction. In this case, however, we are determining what the observation should be, i. e., how high the telescope should be elevated in order to see the sun at a known angular elevation above or below the equator.

In other words, in reducing apparent to true phenomena, refraction is to be subtracted; but in calculating apparent positions from true altitudes, refraction correction is to be added.

The tabular corrections for refraction are exact for the middle day of the five-day period into which the Ephemeris Tables are divided. For extreme days of any such period, an interpolation can be made if desired.

Latitude Coefficients for Refraction Correction

Lat.	Coeff.	Lat.	Coeff.	Lat.	Coeff.
15°	.30	31°	.68	47°	1.29
16	.32	32	.71	48	1.33
17	.34	33	.75	49	1.38
18	.36	34	.78	50	1.42
19	.38	35	.82	51	1.47
20	.40	36	.85	52	1.53
21	.42	37	.89	53	1.58
22	.44	38	.92	54	1.64
23	.46	39	.96	55	1.70
24	.48	40	1.00	56	1.76
25	.50	41	1.04	57	1.82
26	.53	42	1.08	58	1.88
27	.56	43	1.12	59	1.94
28	.59	44	1.16	60	2.00
29	.62	45	1.20	61	2.06
30	.65	46	1.24	62	2.12

Directions for Using the Solar Attachment

Prepare a table of declination values, corrected for refraction, for the day of observation, showing the settings for each hour as directed in the pages preceeding and in the insert appendix in the back of this publication.

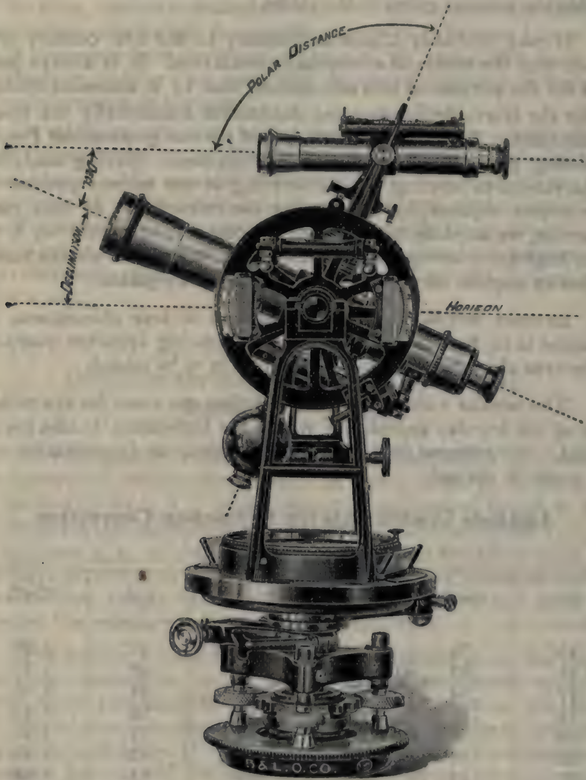


Fig. 94

The procedure described in the examples given should be carefully followed. The hourly change in declination is never quite one minute of arc, so that a setting prepared for 3 p. m., for instance, will be good between 2:45 and 3:15 o'clock.

1st step. Bring the sight line of the solar to revolve in the same vertical plane with the main telescope by sighting both at some distant point. Incline the transit telescope until the amount

of declination, for any chosen hour of observation, is indicated on the vertical circle. If the sun's declination is north, depress the transit telescope; if south, elevate it. Reference to the diagram on p. 160 will suggest the reason for this.

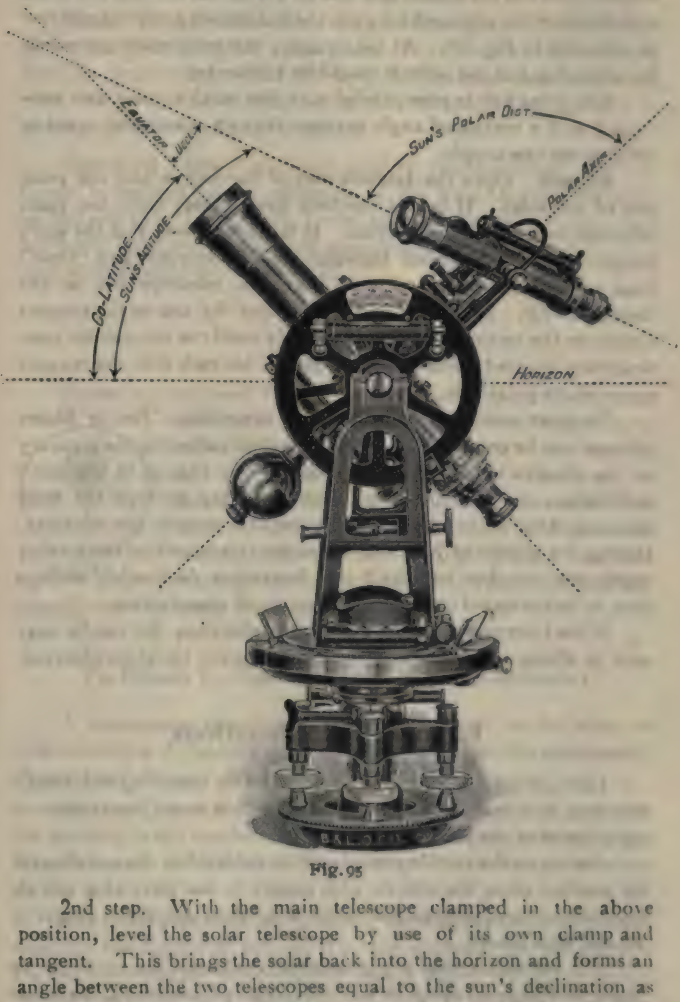


Fig. 95

2nd step. With the main telescope clamped in the above position, level the solar telescope by use of its own clamp and tangent. This brings the solar back into the horizon and forms an angle between the two telescopes equal to the sun's declination as shown in the engraving, Fig. 94. The angle between the polar axis

and solar telescope is equal to 90° plus or minus the declination, or the sun's polar distance.

3rd step. Without disturbing this relative position of the telescopes, open the horizontal axis and set off on the vertical circle the co latitude of the place, which gives the instrument a relative position as indicated in Fig. 95. At the equator, the main telescope would be vertical and at the poles it would be horizontal.

The polar axis is now parallel with the earth's polar axis prolonged, and a horizontal angle between the two telescopes, equal to ZPS, is the one sought.

4th step. Open the azimuth axis of the transit and the polar axis of the solar. If in the northern hemisphere, point the main telescope in a southerly direction. It is evident now that the sun's image cannot be centered between the "equatorial" and "time" lines in the solar telescope until the main telescope lies in the meridian, ZP. Maneuver the instrument by use of the tangent screws on the vertical and polar axes only until the sun can be continuously bisected while following him in his path with the tangent screw of the polar axis.

No great speed is required in this operation. Ten or fifteen minutes can be consumed if necessary without affecting the accuracy of the observation. If there was no hourly change in the sun's declination, the sun could be followed all day through the solar telescope if the main telescope and polar axis were in the meridian. During the middle of June and September this condition can be very nearly realized, but in March and September declination settings have to be corrected carefully for each set of observations.

If the instrument is provided with a compass, the needle may now be allowed to swing freely and the magnetic variation observed.

Latitude Determinations

Latitude may be variously determined by observing the transit of a star, by a mean altitude of polaris or by a direct observation on the altitude of the sun at apparent noon.

Owing to the earth's annual motion in its orbit, the sun changes his position along the ecliptic with respect to the stars at a not altogether uniform rate; so that some solar days are either longer or shorter than others.

For the reason that a chronometer could not conveniently be made to change its speed to suit this solar phenomenon, there has been established a uniform system of time called "mean solar time."

The difference between mean noon, when the sun should be on the meridian, and apparent noon when the sun actually is on the meridian, is called the "Equation of Time." The tabular corrections will be found in the Ephemeris Tables.

Thus, in early November the sun has passed the meridian more than 16 min. before mean noon. It is always well to begin latitude observations some 20 min. before local noon, although there will be seasons of the year when the sun will not attain its greatest altitude until after local noon.

Standard Time * will also qualify the argument, but this should be studied out by reference to the map inserted between pp. 163 and 164. In Western Texas, for instance, observations need not begin until nearly 1 o'clock standard time; whereas in Erie, Pa., they should begin shortly after 11.

Procedure

Follow up the lower limb of the sun, and when the maximum altitude is found add the sun's semi-diameter, as taken from the Ephemeris Tables, to the reading on the vertical circle; subtract correction for atmospheric refraction, as figured by interpolation from the table, p. XXI, and correct this result by the sun's declination: adding if south and subtracting if north. The final result is the co-latitude or the polar distance.

Fig. 96

To Obtain Latitude with the Solar Attachment

A mechanical correction for the declination can be made as directed above if the observation is made with the solar attachment. Proceed as follows:

Level the instrument carefully, point the main telescope toward the south if in the northern hemisphere and bring the solar telescope into the same vertical plane by sighting both telescopes at some distant point.

If the sun's declination is south, elevate the telescope in the northern hemisphere or depress it in the southern hemisphere. If the sun's declination is north, reverse the order. With the main

* *Adopted by the railroads of U. S. and Canada, Nov. 18, 1883.*

telescope inclined to the declination, level the solar telescope and clamp it. Follow the sun in his upward path, through the solar telescope with the tangent screws of the transit, by keeping his image centered between the two outside horizontal cross wires. This will be most conveniently accomplished by setting up the transit to begin with so that the slow motion of the telescope can be maneuvered with the right hand and the lower slow motion of the vertical axis with the left.

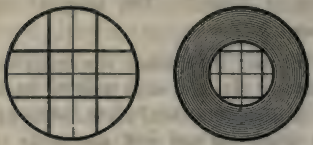


Fig. 97

At the instant when the sun has apparently ceased to rise, take the reading on the vertical circle and subtract the correction for refraction due to altitude as given in the table referred to below. The result is the co-latitude or the angular distance of the equator from the horizon as will be noted by reference to the diagram, p. 160. This is the value required in determining the meridian with the solar attachment but if the latitude of the place is also desired, it can be found by subtracting this net result from 90°.

In his original experiments with the Saegmuller Solar Attachment in 1885, the late lamented Prof. J. B. Johnson said: "It is evident that if the instrument is out of adjustment, the latitude found by a meridian observation will be in error; but if this observed latitude be used in setting off the co-latitude, the instrumental error is eliminated. Therefore, in a meridian observation, always use for the co-latitude that given by the instrument itself."

A Table of Mean Refraction is given on p. XXI. It is the one compiled by Hayford from data furnished by Laplace, Bessel and Doolittle. The table is calculated for the barometer at 29.9 in. (760 mm) and the thermometer at 50° F or 10° C. When the barometer rises to 30.9 in., the factor 1.033 should be used and when it drops to 28.9 in., use 0.966; or for 27.9 in., use 0.933. Other positions will be nearly in proportion. When the thermometer rises to 60° F or 15.°6 C, use 0.981; for 70° F, or 21°.1 C, use 0.962; for 40° F or 4°.4 C, use 1.020, and other situations proportionately. Refraction is *minus* if a deduction is to be made from an observation as in leveling, but *plus* if a setting is to be made for locating a celestial body as in solar work. It has no effect on azimuth.

The change in the sun's position with respect to the stars as already noted, takes place along the ecliptic which is inclined to the equator at an angle of about 23° 27′, intersecting it at the vernal and the autumnal equinox. The result is that any point on the surface of the earth makes one less annual passage under the sun than under a given star. The number of sidereal days in a year is therefore one more than the number of solar days to which we are accustomed; or a sidereal unit of time is of shorter duration than a solar unit in the ratio of 3 min. 56.55 sec. per diem, *mean* time.

Sidereal and Mean Solar Time are identical at Greenwich noon, Mar. 22nd, each year. To roughly translate solar to sidereal time, add 2 hrs. per mo. thereafter and allow 4 min. per diem, thus: on June 7th at noon we have 2 full months and 16 extra days or 5:04 p. m.

Owing to the irregularity, of the sun's motion, the variable length of apparent solar days are equalized for our convenience by the adoption of a uniform system of time measurement called:

Mean Solar Time

The zero point of this system is an imaginary body called the "mean sun" which is supposed to move uniformly along the equator keeping as nearly in the same right ascension with the actual position of the sun as is consistent with perfect uniformity of motion. *

The mean solar time is the hour angle, or the right ascension of the imaginary mean sun and differs numerically by the difference of the hour angles or the Equation of Time.

To Convert Apparent Time into Mean Time, consult the ephemeris tables and select the tabular correction for the day of the month upon which the observation is made. Where the column is headed, "substract from mean time" it should also be construed as "add to apparent time" and vice versa.

To Obtain Time with the Solar Attachment

When an observation for time is to be made, proceed with the meridian determination as previously directed, but at the instant the sun is finally centered in the (clamped) solar telescope, note the local time to the nearest second on some reliable time piece.

Consult "Field Astronomy for Engineers" Geo. C. Cowstock, Univ. of Wis., 1910, p. 39, etc.

Turn the main telescope downward and locate the meridian on a stake without disturbing the position of the Solar. Bring the main telescope back to a horizontal position and clamp. This secures a truly vertical position for the polar axis.

Turn the solar telescope downward and fix a stake in any convenient place as indicated by the central vertical wire. The angle subtended at the instrument between these two stakes is the sun's right ascension, or hour angle, for the instant of observation. Measure as accurately as the graduation of the instrument will permit and reduce the angle read to time, remembering that 15° = 1h; 1° = 4 min.; 15′ = 1 min., and 1′ = 4 sec.

Example

Observed Azimuth	49° 38′ 30″
3 hours	45°
	4° 38′ 30″
16 min. =	4°
	38′ 30″
2 min. =	30′
	8′ 30″
30 sec =	7′ 30″
4 sec =	1′
3 h. 18 min. 34 sec.	

See Table p. XXIII, Appendix, taken from Genl. Land Office Field Manual

Sun's Parallax in Altitude

Lat.	Par.	Lat.	Par.	Lat.	Par.	Lat.	Par.
0°	8.94″	25°	7.99″	50°	5.67″	75°	2.29″
5	8.79	30	7.64	55	5.06	80	1.54
10	8.69	35	7.22	60	4.41	85	0.78
15	8.52	40	6.76	65	3.73	90	0.00
20	8.29	45	6.23	70	3.02		

WHERE a mine is entered by a tunnel or where the ore occurs in slopes not over about 60°, the ordinary transit with legible graduations and proper protection against dust or moisture will answer every purpose; but when the inclination of the sight cuts through the horizontal plates, some provision must be devised for auxiliary sighting in the observation of both precipitous horizontal and heavy vertical angles.

I. If *horizontal* angles are to be read between points in great declivity below the instrument or if alignments are to be prolonged in very steep inclined shafts, the auxiliary telescope, through

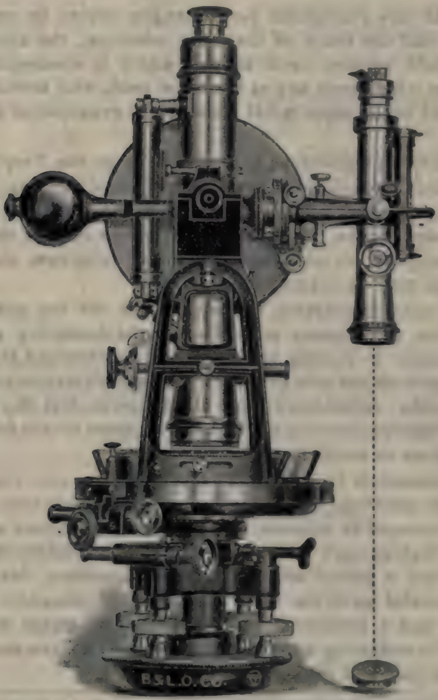


Fig. 98—No. 57 5-in. Tachymeter with Edge Graduated Vertical Circle Equipped to Receive Auxiliary Telescope or Solar Attachment Interchangeably on Same Swivel Mounting

which the points are to be observed, must obviously revolve in the same vertical plane with the main telescope.

For the observation of points in extreme elevations above the instrument, the eyepiece prism, which may be applied interchangeably between the solar, the auxiliary and the main telescope, will answer most requirements. The eyepiece prism is shown applied to the solar in the cut on the preceeding page, also to the main telescope of the 4½-in. Mining Tachymeter as shown on p. 186. For the observation of points, however, in acclivities that closely approach the zenith, the Duplex Diagonal Eyepiece, as shown in connection with the alidade, p. 46, is probably the most serviceable attachment devised for this purpose. With it, zenith observations may be taken and immediately transferred to the plane of the instrument by sighting through the direct ocular with which the device is also provided.

II. If very precipitous *vertical* angles are to be read between points beyond the reach of the main telescope, the auxiliary telescope should revolve upon an axis which is concentric with the circle upon which such angles are to be read, and should therefore be mounted in the same radial plane at one extremity of the horizontal axis.

These two propositions have given rise to the "top" auxiliary and the "side" auxiliary telescope for collateral sighting, which have been variously designed in size and method of attachment since their introduction in 1855*. If, however, a mining theodolite is provided with an interchangeable auxiliary telescope, all errors of eccentricity may be overcome.

The solar attachment corresponds to the top auxiliary telescope. The adjustments prescribed in the foregoing pages cover every necessity in mining work. It will be observed that the precautions set forth not only bring the vertical collimation plane of each telescope into coincidence for all distances, but the spirit level on each telescope provides a ready and accurate means for adjusting the sight lines to parallelism.**

The essentials in a mining theodolite require that stability shall not be sacrificed to lightness, that the construction shall be as nearly waterproof as possible, the graduations accurate and legible, standards high enough to permit a complete revolution of the main telescope and a partial revolution of the auxiliary, inverting telescopes powerful up to the limit where the illumination is sensibly impaired, and a horizontal axis of the most stable construction capable of transferring a meridian accurately between stations close together in plan and distant in elevation.

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* *See articles by Dunbar D. Scott in Trans. A. I. M. E., Vols. 28, 29, etc. also B. & L. O. Co. Metro III Catalog pp. 22, 52, 53 and 57.*

** *Theory and Practice of Surveying, Johnson-Smith, 17th Ed., 1911, p. 414; also Mine Surveying, E. B. Durham, 1913, p. 181.*

The solar telescope is mounted with a swivel adapter on a carefully centered "solar table" that is ordinarily covered with a centering target for overhead plumbing. The solar spirit level insures parallelism of collateral sighting, and the clamp-and-tangent on the polar axis is utilized to range the auxiliary sight-line into the same vertical plane with the main telescope. The auxiliary sight-line therefore, revolves in a vertical plane that is normal to the vertical axis and azimuth circle so that no correction need be made for angles observed on the horizontal plates. These conditions are guaranteed.

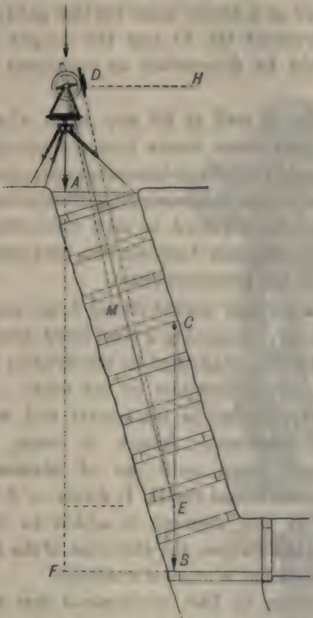


Fig. 99

To Correct Vertical Angles for Eccentricity of Collateral Sighting

In Fig. 99 let the center of the instrument, O, be set up over A at the upper level, and let the auxiliary sight be taken to a nail tied into a plumb line a certain known distance below the hanging point, C, and above the point in the foot plate, B.

The coördinates sought are OF, or AF, and FB. The vertical angle read is HOM, which is too large. The angle sought is HOE.

Then HOE = HOM — OED, or EOM, but *sin* OED (ODE being a right-angle) = OD ÷ OE.

Assuming a concrete example:

Let HOM	(angle read)		= 71°08′
Let OE	(measured dist.)		= 76.09 ft.
Let OD	(eccentricity)		= .315 ft.
Then log.	.315 =	$\bar{1}.498311$	
a. c. log.	76.09 =	8.118672	
		7.616983	
		7.609853	= 14′
	536.4)	7130	= 13″

then 71° 08′ 00″ —14′ 13″ = 70°53′47″

For work of this nature, where the elevation of the station in the foot plate at each level is to be carefully determined as a datum point for that gallery, the height of instrument (H. I.) and the height of point (H. P.) should be determined as suggested in Fig. 99.

For this purpose, as well as for any other where the height of the instrument enters into the calculation, we have designed a height-of-instrument plummet (H. I. P.) as shown in the accompanying illustration, which is to be carefully adjusted to each instrument by allowing for the distance between the horizontal axis and the hook in the plummet chain.

It consists of a circular nickel-plated brass case, $2\frac{1}{8}$ inches in diameter, containing a steel band somewhat over 8 ft. in length, graduated to hundredths of a foot on one side and millimeters on the other. It can be conveniently carried in the pocket and will serve other useful purposes. Attach to hook of plummet chain, pull down until tip of plummet touches the station, then read the H. I. direct — 2.32 as in the figure. If this amount is added to the known elevation of the station, the elevation of the instrument above the datum is at once available.

Pence & Ketchum (p. 140) recommend that for the rapid determination of H. I. in stadia work, the tripod leg should be graduated by experiment, so that when the plumb bob is swung out against it, the amount will be indicated at the point; but the means here described is more direct, convenient and certain.

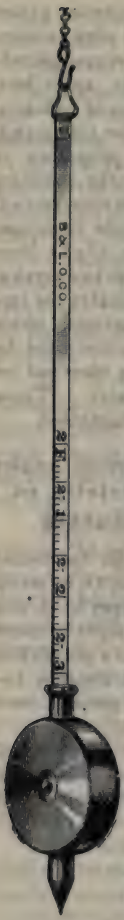


Fig 100

The Tunnel Trivet is a rigid ribbed support with a neck high enough to be easily grasped for carrying, or for suspending a plumb line beneath the instrument when overhead plumbing is not desirable or not convenient. It is a very substantial brass structure as shown in connection with Fig. 101.

The Lateral Adjuster, as shown also in connection with Fig. 101, is used for the final accurate placement of the theodolite when ranging the sight line into an alignment indicated by two wire plumb lines suspended in a vertical shaft. The feed screw of this apparatus has a pitch of 100 threads per ft., and the knurled head is provided with ten points in order that intervals of $\frac{1}{1000}$ ft. can

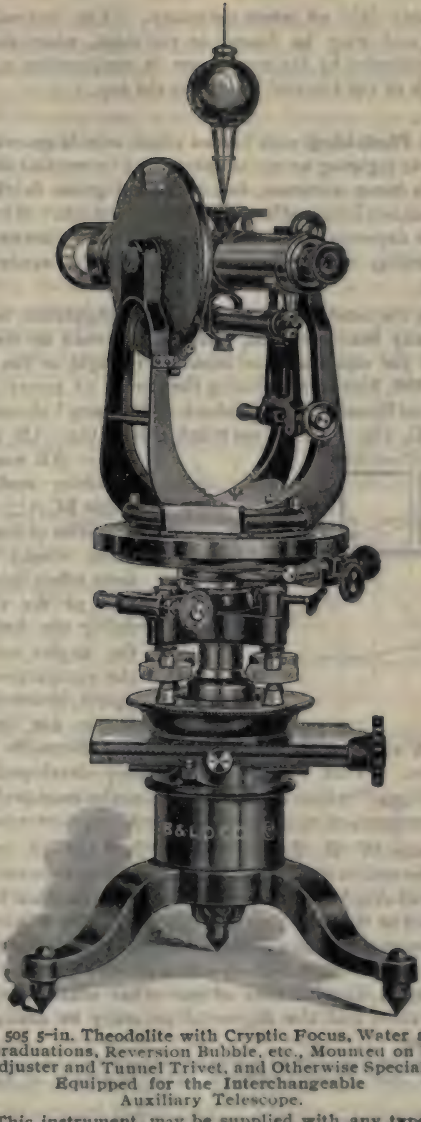


Fig. 101—No. 505 5-in. Theodolite with Cryptic Focus, Water and Tarnish-proof Graduations, Reversion Bubble, etc., Mounted on Lateral Adjuster and Tunnel Trivet, and Otherwise Specially Equipped for the Interchangeable Auxiliary Telescope.

Note:—This instrument may be supplied with any type of Vertical Circle or provided with a Diagonal Eyepiece or a Striding Level, etc.

be accurately laid off when necessary. The normal position is indicated, and may be found in the dark, when the ribs in the position indicated by the arrows are in conjunction and when the index tooth of the knurled head is at the top.

Shaft Plumbing with loaded piano wire is generally preferred to telescopic sighting or optical plumbing in vertical shafts; not because it is more accurate, but the usual great depths and dense atmospheres are inimicable to the latter methods. The accuracy of orientation depends somewhat upon the distance between the wires and the facility with which the theodolite is ranged into their alignment.

When the conditions are such that triangulation is necessary, it has generally been assumed, without reference to the purpose in hand, that the equilateral or isosceles triangle is the best shape. Prior to 1850, however, utilizing the theory of errors as developed by Gauss and Bessel, Prof. Weisbach of Freiberg* showed that when the side BC, Fig. 102, was not much greater than AB, in fact, when BC ÷ AB was as small as possible, and when the angle BCA $\leq 25'$, that the crudities of linear measurement would produce the least effect upon the solution of the triangle. In fixing the limit of errors for angles and distances, the comparison should, no doubt, be consistent. There is no use in measuring angles with extreme accuracy and adjacent distances by close approximation. This theory is elaborately developed in a recent work by Briggs ** which doubtless contains the most profound mathematical treatment of mine surveying problems now available.

Fig. 102

Professor W. H. Rayner of the Univ. of Ill. has also contributed the results of a scholarly investigation into the "Allowable Use of Small Angles in Surveying." We reproduce his argument as published in the *Eng. Rec.* Oct. 18, 1913. He says:

"Every engineer and surveyor knows that for accurate results he cannot rely on computations which involve the sines and tangents of small angles, but his idea of just what relation exists between these functions and the degree of accuracy he desires is usually rather vague.

* *Die Neue Markscheidekunst, Julius Weisbach, 1st Edn., 1850; 2nd. Edn., 1859.*

** *The Effect of Errors in Surveying, Henry Briggs, Heriot Watt College, Edinburgh, 1912.*

"Obviously the reliability to be placed in any values dependent on the measuremen of angles is determined by the accuracy with which the angles are measured. Under ordinary conditions, with a transit or sextant reading directly to minutes, it may be assumed that angles will be measured with a probable error of about 30 seconds, and the accompanying curves have been drawn on that basis. For other degrees of accuracy a direct proportion obtains and results may be derived from the curves. Curves have been drawn only for the functions of sines and tangents, but their adaptability to cosines and cotangents will readily be seen.

"An example or two will illustrate the use of the curves. Suppose it has been decided that in a given traverse or system of triangulation the ratio of error must not exceed $\frac{1}{2500}$, which corresponds to an accuracy of 0.04 per cent. The horizontal line which corresponds to this ratio intersects the ratio curve for tangents

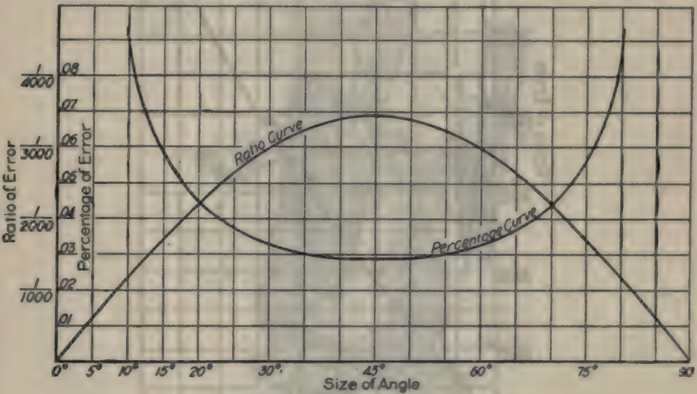


Fig. 103

(Fig. 103) at two points whose abscissæ are 23 and 67 deg. respectively. Hence in this survey, if computations involving tangents are to be made, the angles must lie between these limitations. The percentage curve may be used in a similar manner. When sines of angles are involved, the lower limit is the determining factor, since the value of the sine changes rapidly only for small angles, and Fig. 104 may be used in a similar manner. In the case given above, the lower limit would be 20 deg.

"If it were impracticable, however, to limit the size of angles to 23 and 67 deg., then an instrument reading to 20 seconds, or the method of reading angles by repetition, would be necessary. Suppose, then, by one of these means the probable error in reading angles were reduced to 15 seconds, or half the former value. The curve may then be adapted to this case by changing the numerical values of the ordinates to half their value, or by dropping the line

to the indicated value of $\frac{1}{1250}$. This line intersects the ratio curve (Fig. 103) at the points whose abscissæ are 11 and 79 deg., and accordingly these angles would constitute the limits for a ratio of error of $\frac{1}{2500}$ for angles measured with a probable error of 15 sec. Referring to the sine curves it is seen that the lower limit in this case would be 10 deg.

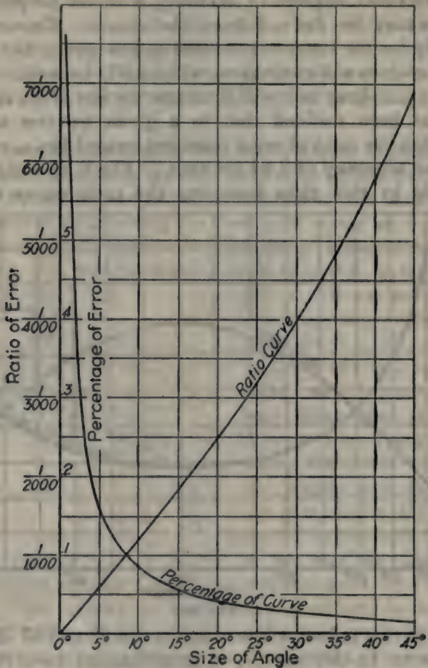


Fig. 104

"In systems of triangulation such as those carried out by the U. S. Geological Survey, the lower limit of the size of angles is often taken as 30 deg. Let us see to what ratio of error this corresponds. From Fig. 103 it is seen that the ordinate for the angles 30 and 60 deg. equals a ratio of $\frac{1}{3000}$, but this applies to angles measured with a probable error of 30 sec. In such work as has been referred to, the probable error of reading angles is reduced to about half a second, or, in other words, about sixty times the accuracy assumed in the curves shown is obtained. Hence the ratio of error would equal about one-sixtieth of $\frac{1}{3000}$, or $\frac{1}{180,000}$. From the sine curve this ratio would be $\frac{1}{240,000}$.

"Of course, in all these cases, the maximum error has been under consideration. The average error in computations would be much less, although for strict consistency of results the maximum error is the controlling factor.

"These curves should be useful to the surveyor or engineer in enabling him to decide quickly and accurately the limiting size of angles if he desires consistently accurate results, and whether or not more refined instruments or methods of measuring angles will be necessary under any given conditions. Other uses of the curves will be evident in any connection where computed results depend on measured angles."

The Interchangeable Auxiliary Telescope

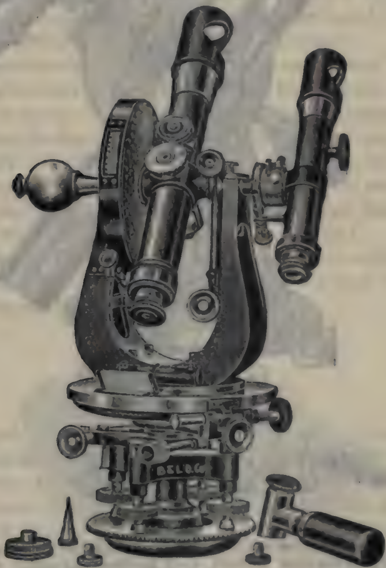


Fig. 105—No. 457 4½-inch Theodolite with Edge Graduated Vertical Circle Showing Electric Hand Magnifier, also Interchangeable Auxiliary Telescope Applied to Side of Instrument.

The interchangeability of the auxiliary telescope has greatly improved and facilitated mine surveys by producing a single positive result from a double negative condition. With the ordinary fixed top or side telescope, great importance has been justly attached to parallelism of collateral sighting; but in the case of the interchangeable auxiliary, if the adjustment for alignment is maintained for all

distances, parallelism between the sight lines, as well as the exact amount of eccentricity, is of no material consequence if the conditions expressed in the propositions on pp. 173-4 are observed.

With the **auxiliary applied to the side** as in Fig. 105, *vertical* angles may be observed without correction. They should preferably be observed directly with the auxiliary first applied to the right, then with the instrument reversed, the auxiliary being at the left. If there is a discrepancy, the mean reading will be exact within a very small correction which is insignificant.

Fig. 106—No. 47 4½-inch Tachymeter with Edge Graduated Vertical Circle, Reversion Bubble and Interchangeable Auxiliary Telescope Mounted for Mining and Slope Stake Work.

Obviously such vertical angles should be determined with care, for small errors have an appreciable effect in resolving the measured distance into its vertical and horizontal components. Horizontal angles, as from a steep shaft alignment into a drift, are to be doubled by necessity—first from the right, then the left—and averaged. The errors of eccentricity and parallelism, whatever their amounts, will be neutralized, and this process of reversion also tends to correct errors in the instrument itself.

When the auxiliary is applied to the top of the instrument, it is securely mounted on the same "solar table" with swivel adapter as supplied for the Solar Attachment. This mechanical principle is nearly if not quite unique and obviates the necessity of fixing a vertical pillar to the top of the main telescope. In this position, *horizontal* angles may be observed without correction if the vertical wires are ranged into the same alignment. The adjustment for parallelism is not necessary, but if the operator prefers to secure this relationship he may follow the directions under the head of Adjustments. The least accessible station may be sighted with the auxiliary, and the other with the main telescope. The eyepiece prism may be used interchangeably when the circumstances require it. The collimation being correct in both telescopes for all distances, the auxiliary may be mounted with its eyepiece towards the main objective, but must be adjusted in alignment against some previously established test line, which limits this practice to rare occasions.

Fig. 106 shows the individual axis of revolution with which our auxiliary telescopes are equipped. The little clamp-and-tangent is used to range the auxiliary into alignment with the main telescope. The illustration shows the auxiliary turned off to one side as in slope-stake work, which indicates the possibility also of setting the auxiliary at exactly 90°, or at any desired angle with the main telescope for city surveys or any such work where fixed deflections can be established while the main telescope is horizontal.

The scheme is also recommended for tracing concealed outcrops on hillsides, if the auxiliary can be set to the dip and strike of the vein at any exposed portion.

Slope Stake Setting

In setting slope pegs for railway, canal or highway embankments or excavations by the usual method of trial, several approximate calculations have to be made before the exact position of the peg can be found. The labor itself is not difficult, but the time required is such that, if the work is done faithfully, not over about 50 stations per day can be accomplished. Allowing for intermediates and grade points, this is equal to about half a mile. Single cuts or fills are occasionally of this length.

By employing this special mechanical adaptation of the auxiliary telescope, as introduced by us in 1912, it is only necessary to put in guide pegs at the center and termini of a fill or a cut, and range all intermediate pegs without physical or mental exertion as fast as the axe-man may be able to set them.

Refering to the profile, set guide stakes at A, D and G by the usual trial method, using the telescope bubble of the theodolite. Place the instrument back on the hillside squarely behind the central

point chosen to command a view toward both ends of the cut. The telescopes being parallel and in the same vertical plane, set the vertical limb to the angular value of the slope, as suggested in the lower cut of Fig. 107. Move the instrument forward or backward until the horizontal wire of the auxiliary cuts the ground at the central guide peg, D. Revolve the auxiliary on its own axis, all other clamps being set, and test the sight against the extreme guide pegs. A little maneuvering, perhaps by turning the whole instrument on the vertical axis, will presently get the auxiliary revolving in the direction and inclination of the slope, and in this position the intersection of the sight line and the ground will mark the surface horizon of the cut.

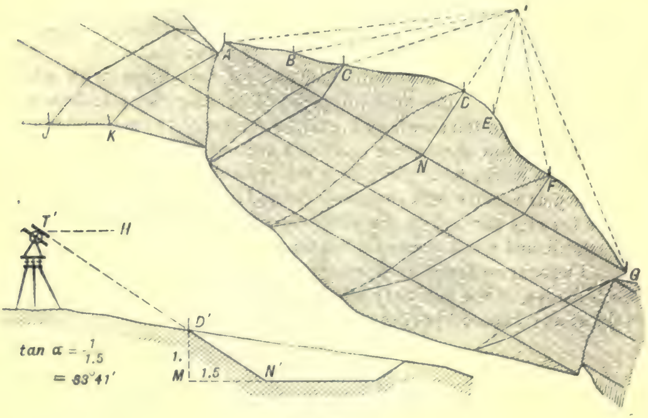


Fig. 107

The pegs set, elevations for earthwork can be taken at leisure. These rules apply only to tangents on which the grade is uniform. At each change of grade a fresh setting must be made. The appliance may be used with equal facility for embankments. Following the above suggestions the *modus operandi* will suggest itself to the surveyor. If the ground is not visible at any point, a rod should be held inclined until the upper portion coincides with the "horizontal" wire of the auxiliary. The origin of this idea can be definitely located in England in 1864 when James Lister, C. E., had the cradle theodolite remodeled for this purpose.* He arranged a method also of setting slope stakes around a curve with this device, but the process is rather involved and probably would not be adopted if demonstrated in this text.

* *Surveying Instruments, W. F. Stanley, London, 1895, p. 357.*

Parallelism of the auxiliary sight line is to be secured when attached to the top by firmly fastening the swivel base to the "solar table" so that no lost motion is apparent. Measure the distance between the centers of the telescopes by taking a mean between the inside and outside distances between the tubes. Mark off this amount on a sheet of paper at, say, 100 ft. from the instrument, so placed that the lower mark may be seen while the main telescope is in a horizontal position. If the auxiliary does not cut the upper mark, turn the worm in the hub, as shown in Figs. 106 and 107, until a coincidence is perfected. Turning the worm screw tends to turn the telescope slightly in azimuth, but this can be corrected by the tangent screw just above. In our latest device for securing parallelism, the worm thread and the steady pin mentioned have been superseded by opposing capstan head screws in the adjusting block just under, and in the line of, the auxiliary telescope. These screws simply regulate the position of the encasement which contains the axis of the auxiliary. One is marked with an **X**. If this is always set toward the ocular a uniformity of seating will be guaranteed. To adjust the auxiliary then against the test mark, simply turn the capstan studs in opposite directions and be certain to take up all lost motion.

When the auxiliary leaves our works, the cross wires are carefully collimated to the optical axis and should never thereafter be molested, because the expedient of moving the cross wires in the test for parallelism is both theoretically and practically incorrect. The analogy between this argument and that on the adjustment of the dumpy level is marked. If one wishes to verify the collimation adjustment, he may do so by revolving the auxiliary longitudinally in a set of improvised wyes. The outside of the tube is not necessarily concentric with the axis of the objective to the utmost degree of precision, but the method is superior to any yet devised.

The test for parallelism is in reality desirable only for slope stake setting and unless the surveyor insists on reading *vertical* angles with the auxiliary at the *top*, the adjustment is not indispensable for mining work. If it is thought desirable for any reason to secure parallelism when the auxiliary is applied to the *side*, it may be accomplished by similar means.

Unless the surveyor wilfully disregards the purpose of the principle of interchangeability as laid down on pp. 173-4, there is no occasion for deep concern about any adjustments in the auxiliary telescope for mining work except those of alignment, centering and verticality of the vertical wire.

The Alignment Adjustment is one which requires only a simple mechanical manipulation just before each set of observations. The adjustment is not a permanent one. If the auxiliary is at the

Fig. 108
No. 47, 4½-inch
Mountain
and
Mining
Tachymeter
mounted on
Reducing Ring
and short leg
Tripod
for work in
"Scrams"
or
Shallow Coal
Measures.

top, sight some object or candle light within convenient range, with the vertical wire of the main telescope, all clamps set. While sighting through the auxiliary, bring its vertical wire to bear on the same point with the small clamp-and-tangent which operates on its mechanical axis. If at the side, the same general procedure is followed, but what was the vertical wire at the top has become the horizontal wire at the side. One wire then in the auxiliary, properly collimated to the equator of the field, would meet every requirement for mine surveys.

To attach the Auxiliary to the side of the instrument, unscrew the swivel distance piece which is used when applied at the top. This reveals a threaded hub which may be securely screwed into either socket at the extremities of the horizontal axis. For reading vertical angles in this position it is not necessary to attempt to carry the instrument into the mine after having adjusted the side telescope for parallelism at the surface.

Centering is accomplished by the maker by so placing the mount that, when the bulls-eye target is screwed on, it may be used for overhead plumbing as in Fig. 101. This test, however, may be conducted precisely as designated for the solar attachment, (see p. 163). Importance of centering for very steep sights is manifest. An eccentricity of $\frac{1}{100}$ of an inch on a horizontal sight of 100 ft. is a negligible quantity, but when the inclination of the sight reaches 75°, or thereabouts, the horizontal component is reduced to 25 ft. and the error is increased to about seven seconds.

The Illuminator Tube

When looking into a dense background, as in mining or stellar observations, there is scarcely ever sufficient illumination about the object sighted to make the cross wires visible. In cases

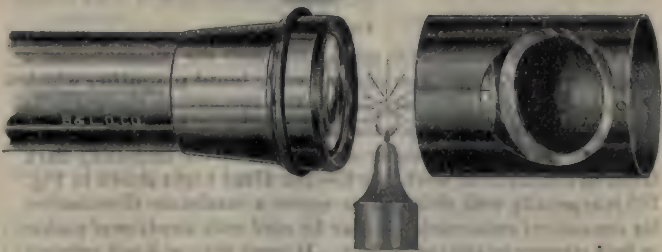


Fig. 109

of this kind a convenient and satisfactory means of illumination is secured by projecting a little diffused light back into the telescope tube by means of a reflector ring that is inserted diagonally in a tube, as suggested in Fig. 109. See also Figs. 105 and 108.

The nickel-plated ring will catch enough light to accomplish the desired result. It is neither necessary nor desirable that the reflected light shall be concentrated at the diaphragm. Such a plan would so flood the field that the dimly visible object would be obscured. The reflector ring is made narrow partially on this account and for the further desirable purpose of keeping the aperture and light gathering capacity of the objective as large as possible. A small opening defeats the purpose for which the larger apertures are calculated. (See pp. 78 and 79).

The reflector ring can be removed if desired, and by turning the opening downward in an emergency the tube can be used as an ordinary sunshade.

Electric Head Light

Artificial illumination of the diaphragm may also be accomplished by perforation of the horizontal axis, as described on p. 144. The source of light is represented there by a candle but the larger theodolites are usually equipped with a bronze lamp mounted on a bracket. In more recent years the dry battery electric bulb lamp has come into favor on account of its relatively greater luminosity and cleanliness.

Fig. 110

On page 181 we show in connection with the illustration of a small mining theodolite, that of an **electric hand magnifier** which could be used not only for reading the verniers in the dark, as originally intended, but for axial illumination as described.

For mining engineering or rescue work or for inspecting dark forms in construction work, the Electric Head Light shown in Fig. 110 is especially well designed to secure a maximum illumination for the current consumed. It may be used with any type of pocket or belt primary or storage battery. If used with a 3-cell primary or 2-cell storage battery, a 4-volt bulb is used; but if with a 3-cell storage battery, a 6-volt lamp is supplied. The burnished silver parabola reflector is insulated against short circuits and will project a beam of light over 100 yds. as well as a large circle of illumination close to the wearer. Batteries burn twelve to twenty hours.

The Short Focus Lens is an additional lens applied to the objective so that the focal length of the combination can be sufficiently reduced to observe objects in cramped places that are very near to the instrument. For this purpose it is presupposed that the collimation adjustment in the theodolite is perfect for all distances—particularly short ones. (See top of pp. 7 and 67).

Fig. 111

It may be rationally assumed that unless some special precautions are observed, the casual application of a second objective lens will create a new optical axis and modify the collimation adjustment. For observing horizontal angles, however, an error in the vertical plane only, as in cut III Fig. 112, would be negligible; but some rapid and accurate method of adjustment must be devised as explained below.

Description	4½ in.	5 in.	6 in.
Shortest sight from objective, **erecting** telescope, ...	44 in.	46 in.	77 in.
Range with S. F. lens, in inches, .	62 to 20	52 to 22	88 to 34
Shortest sight from objective, **inverting** telescope, ...	64 in.	96 in.	
Range with S. F. lens, in inches, .	70 to 30	125 to 45	
with two lenses		to 28	

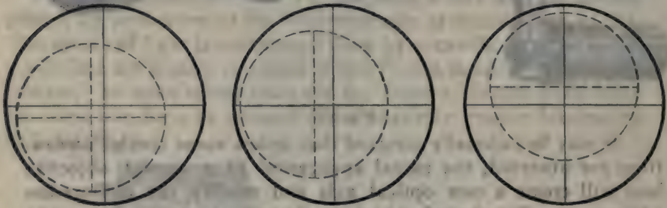
The 4½ and 5-inch erecting telescopes focus normally down to within considerably less than 5 ft. from the objective, as indicated in the reference table. These, or any other telescope, could be made to focus normally upon an object much closer by extending the focusing rack movement; but for mining work particularly there is a limit to such an expedient, fixed by the necessity of transiting the telescope through the standards at the objective end. If one wished to reduce the outer conjugate focal length of a 4½-in. inverting

telescope, below 64 in., as indicated in the table, he would be required to apply the S. F. L. attachment and get control of distances ranging between 70 and 30 inches.

With most S. F. L. attachments heretofore designed, it has not been possible to sight less than 5 ft. with the instrument nor over 4 ft. with the additional lens applied. Under these circumstances, collimation checks are impossible.

The reference table gives for the 4½ and 5-inch Tachymeters and Theodolites, minimum sights for the instrument and maximum and minimum length of sights with the S. F. L. applied. The longest sight with the secondary lens overlaps the shortest sight with the instrument so that for average conditions, only one extra lens need be employed, except in the case of the 5-in. inverting telescope where two lenses are required to reduce the sight to 28 in.

Let it be assumed that there exists some eccentricity. The very fact that there is, will provide a ready means of adjusting the



I II III

Fig. 112

lens to the required purpose. In the position I, both wires are assumed to be displaced, but by continued rotation a correction can be made for the horizontal or vertical wires as required. The S. F. L. will rarely be used for leveling purposes as in position II. Sight some object, therefore, with the vertical wire of the main telescope at its shortest range and securely set all clamps. Apply the S. F. L.; re-focus the telescope and revolve the lens on the barrel of the objective mount until the vertical wire appears to coincide with the test point, as in cut III. For future use mark a check line against an index on the barrel of the lens with the letter V and apply the supplementary lens thereafter in this position for alignment work or for reading horizontal angles. For leveling purposes the horizontal wire can be checked up in the same manner, as in cut II and the index marked with the letter H. Obviously this will occur at a quadrant distance from the index line V. The argument is not made complicated or difficult with adjusting screws.

In manufacture we are enabled to find a position for the S. F. L. where it will be perfectly centered in both planes and cement it there. The above suggestions are offered only in case of necessity.

Owing to the nature of mining work observations are frequently taken in the vicinity of the zenith or nadir. The ordinary transit instrument can be equipped rather indifferently for this class of work by applying either the ocular prism or diagonal eyepiece to the eye end of the telescope, or a deflection prism to the objective.

The ocular prism or diagonal eyepiece require no more adjustment than the eyepiece itself (see p. 7). After a collimated sight line passes the diaphragm and the intersection of the cross wires has been indelibly stamped, as it were, upon the aerial image of the field, it may be manipulated or deflected in any manner that suits the convenience of the operator. Like the S. F. lens, however, the objective prism requires some special adjustment to harmonize with a sight line already collimated to the optical axis.

The Eyepiece Prism is a total reflecting prism, being one-half of a cube of flint glass cut diagonally across one side. In total reflection there is little loss of light from transmission or absorption and none from refraction so that there is accordingly produced an image of great brilliancy. The field of view, however, is slightly reduced because the eyepoint is necessarily removed from the position of the theoretical exit pupil. The E. P. P. may be used in connection with the erecting telescope but it is more appropriately associated with the inverting type, for the single reflection surface erects the image in the vertical plane; the right and left sides of the field, however, are still reversed and this phenomenon cannot be rectified unless the reflecting surface is roofed. The E. P. P. has a limited range of action because the eyepoint cannot advance beyond the barrier of the standards, say at about 75°. It is used most commonly with the mining theodolites and alidades which have celestial telescopes, or with any instrument adapted for solar observations. In the latter case the swivel ruby glass ray filter shown in the illustration is thrown into the line of sight.

Fig. 113

The Diagonal Eyepiece is indispensable between the limitation just mentioned and the zenith. The optical conditions are such that the power of the inverting telescope, with which it is exclusively associated, is usually increased beyond its normal magnification. We manufacture diagonal eyepieces in two of the latest and most improved designs, as illustrated in Fig. 114. Optically, they consist of an ocular and an erecting system of lenses interrupted by a plain total reflecting prism. The erecting system gives the additional length of tube required to extend the position of the eye-point and incidentally erects the image in both planes.

The Amplex Model shown with a hinge is especially designed so that the telescope may transit at the eye-end while this device is applied, in order to correct for possible errors in the horizontal axis. In connection with our Tachymeters such a mechanical provision is not necessary, for the telescopes transit at the objective end. The focusing is accomplished with the knurled head at the base.

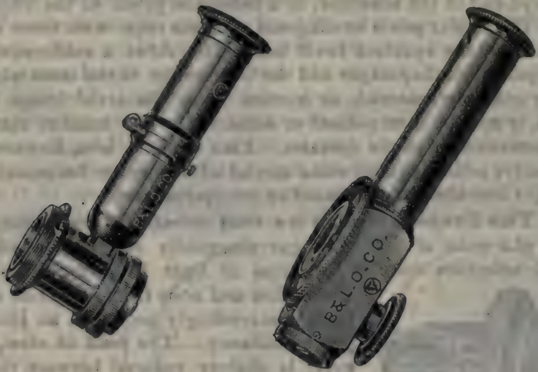


Fig. 114

The Duplex Model is so designed that the tube slides up and down at will between stops on the swivel adapter ring so that the prism may be instantly thrown into, or removed from the sight line as required. This arrangement permits one to transfer sights between the zenith and the horizon with minimum effort and chance of error. This also makes it an ideal construction for the Standard Adjustment as described on p. 72. The large milled swivel head is used to attach the appliance to the telescope without turning the whole device. Some engineers prefer to connect the mine traverse with the surface survey by carrying the reference line upward on account of the advantage of sighting against daylight. If the shaft is dry and upcast, this system possesses sundry advantages.

The Objective Prism of the ordinary variety, as shown in Fig. 115, is now nearly obsolete. It was used by Steinheil for astronomical observations in 1847, and Prof. Stampfer applied it to a wye level in 1852 for nadir sighting in mines. The casual application of such a prism does not guarantee a constant deflection angle, and any error in the relative position of the reflecting surface to the incident beam will be *doubled* in the reflected ray. To illustrate: let ABO, Fig. 116, be the correct position of the prism to project the incident ray, MR, at exact right angles to its original direction;

Fig. 115

but let it be assumed that in applying the prism, the reflecting surface conforms to a deviation of 5° as indicated by the dotted line. A glance at the figure will show that in this case (not allowing for refraction in the glass) the incident ray will be reflected to RN_1 and the included angle will be 100° instead of 90°.

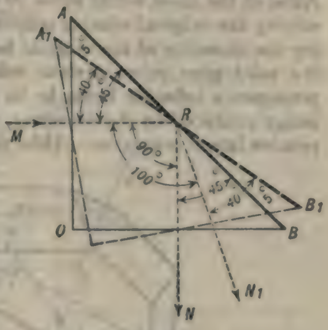


Fig. 116

Probably the first prism to project an incident ray invariably at right angles with its original direction was designed by Dr. H. Wollaston, an English scientist, in 1804 It is commonly known as the "camera lucida" frequently employed for perspective drawing in microscopical examination, but rarely with the transit because of its limited field as compared with its size and weight.

The Wollaston Prism as shown in outline, Fig. 117, is the quarter section of an octagon. The angle of reflection between AB and BC = $ERR_1 = RR_1F = 135°$. If the beam of light can enter the face anywhere between A and G, it will be reflected successively from AB and BC and ultimately pass out between C and H so that FN will be at right angles to the incident beam ME, regardless of slight displacements of the prism in the plane of reflection. If a perpendicular is dropped from B upon the catheti at G and H, there will be defined the limitation of the sides AD and CD to which BA and BC are related. The portion of the prism block, GDH, is therefore of no service and is discarded. If such a prism were to be employed for nadir sighting, or otherwise, the diameter of the face, AG, would have to be somewhat more than the aperture of the objective. If it were practical, its most appropriate application would be in connection with the ocular of an erecting telescope as an eyepiece prism, for the double reflection would preserve the original erect position of the image; but the path of the emergent beam through the prism, would be of such length that the size of the field, as well as the illumination, would be very materially reduced.

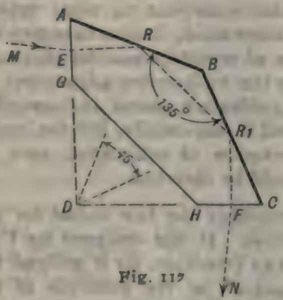


Fig. 117

The Abbe Erecting Prism was designed by Dr. Ernest Abbe of the Zeiss works some 20 years ago for the purpose of preserving the original relative proportions of the field without changing the direction of the original beam.* As shown in Fig. 118, it is a solid glass prism of four reflecting surfaces which not only propagate a reflected beam in its original direction but erect the image produced by the objective, while in the roof is preserved the relative lateral positions. Abbe used it as a substitute for the Porro

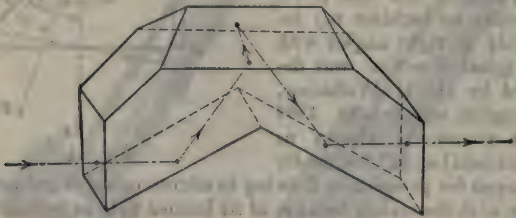


Fig. 118

system of prisms now used in hand binoculars, and it has been used in transit telescopes between the objective and diaphragm as a substitute for the ordinary erecting lens system. Its advantage here lies in the fact that it adds an appreciable increment to the focal length of the objective, contributing also somewhat to other optical qualities in consequence. On the other hand, it absorbs somewhat more light, and studying the effects of remotely possible displacement, its important relationship to the collimation adjustment is apparent. Where the higher magnification seems indispensible with small instruments we have used it successfully, but the image is not quite so crisp and the precaution of collimation tests in marked changes of temperature should be observed.

The Penta Prism is unquestionably the best suited for use in connection with telescope objectives. It was designed by Col. Goulier in 1864 but was really not an invention, for it utilized the basic principle of the "optical square" or the principle of the relationship between the index and horizon mirrors of the sextant. It is a well known fact that when a ray is reflected twice in the same plane, the second reflection makes an angle with the incident ray equal to twice the angle included between the reflecting surfaces. Fig. 119 shows two ray paths being reflected successively between the silvered surfaces with an idea of emphasizing the fact that whatever the original direction of the incident beam, the projected ray will pass out of the penta prism at right angles to that direction if

* *Vereinsblatt der deutschen Gesellschaft fur Mech. u. Optik, 1895, No. 10 Theorie der Moderne Optischen Instrumente, A. Gleichen, 1911, p. 152.*

the angle included between the reflecting surfaces is 45°. Slight movements, therefore, in the plane of reflection will produce no error in the path of the reflected beam.

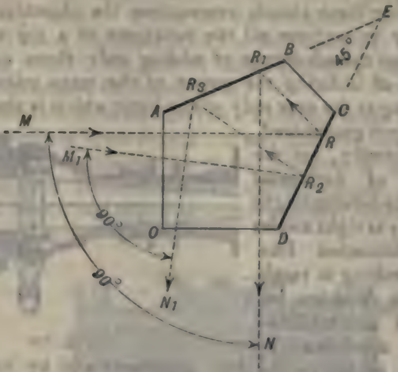


Fig. 119

Fig. 120 shows how the penta prism is mounted in a separate housing with a clamp for securing it to the collar of the objective. It is also provided with an axis of rotation, normal to the telescope, that is provided with a separate small clamp-and-tangent movement. The most reliable results are now possible because of the auto-correction in the prism itself and the constant amount of eccentricity due to the fixed position of the objectives supplied with our Tachymeters and Theodolites. The attachment applied in this way to any ordinary instrument converts it suitably for mining surveys. Vertical angles only would have to be corrected for eccentricity of collateral sighting.

Fig. 120

Adjustment

To use the appliance for the observation of a very precipitous horizonial angle, sight some point with the verticle wire while the telescope is tipped downward, all clamps set. Apply the penta prism to the objective collar and clamp securely. Open the prism-box axis, elevate the telescope (adding if necessary the eyepiece prism) and rotate the objective prism, first by hand then with its own

tangent movement, until the test point is again cut by the vertical wire. The reflecting surfaces now lie normal to the vertical plane of the instrument.

Observing these simple precautions the objective penta prism is now ready to perform all the functions of a top auxiliary telescope. It has the advantage of adding only three ounces to the weight and of utilizing the power of the main telescope for auxiliary sighting at the cost only of the light which is absorbed in the glass. It is not guaranteed, nor is it necessary, for the observation of *horizontal* angles that the deflected sight shall be truly at right angles to the collimation line. Optical plumbing is rarely attempted in America, but it may be accomplished with this appliance by sighting in points on the four quadrants and by locating an intersection between them. Fig. 121 shows the path of the six principal rays passing through the correction wedge, the penta prism and the secondary objective as supplied with our Prismatic Mining Tachymeter. This instrument was designed to utilize the principle of interchangeability between the principal objective and a secondary objective mounted in the outer extension of a perforated horizontal axis.

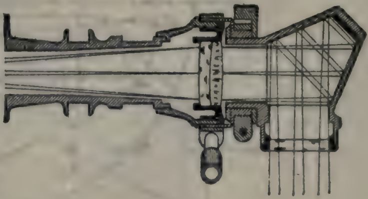


Fig. 121

For the observation of *vertical* angles with the prism mounted as in Fig. 120, a correction for eccentricity would have to be applied (see p. 175), and for this purpose accurate right-angle deflection would have to be accomplished by the correction wedge as above suggested. To overcome this necessity we opened up the horizontal axis and by utilizing the secondary objective referred to above were enabled to increase the power of collateral sighting and use the penta prism with the same facility as an interchangeable auxiliary telescope. The system possesses further advantage in that all observations may be conducted directly from the ocular of the main telescope by utilizing a small interior penta-prism which can be thrown into, or out of, the principal sight line as desired.

Fig. 122

Rapid leveling for setting up in nearly all classes of mountain and mining work is now accomplished quite generally with the extension leg tripod. Those of our manufacture are very substantial, but some engineers object to the possibility of vibration in a tripod leg which must mainly depend upon a pair of clamping bands and set screws for its stability.

To meet this demand the Quick Leveling Head can be used to advantage with the more substantial one-piece leg, but it has not the same capacity for centering the instrument over or under a certain point beyond the limitation of the shifting center. For use with a level and a split leg tripod in rough country, however, this point cannot be raised.

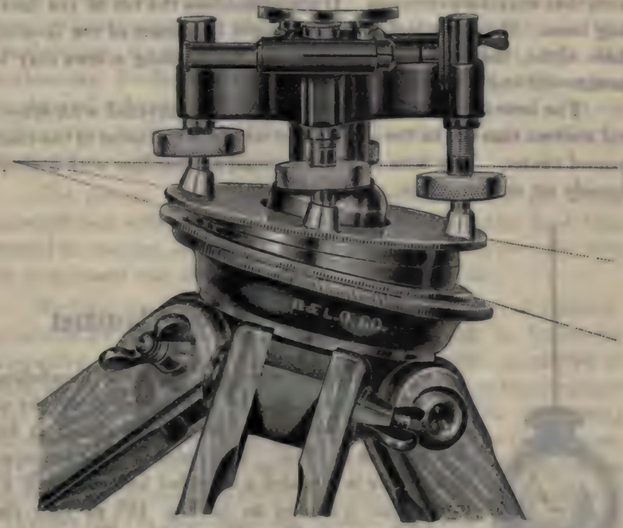


Fig. 123

There have been many types of quick leveling heads designed, mainly on the ball-and-socket principle after the system used for generations with a surveyor's compass, but the center of gravity is too high above the point of support and instability is the inevitable consequence.

This design consists of two wedge shaped discs interposed between the leveling screws and tripod nead. The system sacrifices nothing to stability and preserves all the advantages of the shifting center. The wedge plates may be shifted from side to side by

loosening the leveling screws. To appropriate all the advantages of the Q. L. H., loosen the leveling screws, turn the thickest portion of the lower disc toward the down-hill side; hold fast to this and turn the upper plate, with instrument, until the thickest portions of both discs lie superimposed. Now tighten up the leveling screws and finish the leveling operation with the leveling head, if necessary, as shown in Fig. 123.

The great advantage of this type, particularly when used for transits, lies in the fact that the instrument will not capsize though the screws may be unclamped, and no accident can occur on this account. The illustration shows how the Q. L. H. may be shifted on the tripod head for centering over a peg or under a "spud".

The illustration shows by comparison the amount of inclination that may be overcome. It about doubles the tilt of the leveling base. If greater inclination is sought by means of the leveling base alone, the lower dust guards for the leveling screws may be unscrewed and removed.

The base cups for the leveling screws are provided with spherical sockets that are in the same plane with the ball joint of the compound center. The base cups are secured against loss with a small loosely set anchor-screw.

The Counterpoise Plummet

Fig. 124

The auxiliary telescope for mine theodolites had been counterbalanced merely by a weight of no other utility until we designed a plummet for this purpose in 1910. The point is to be removed when there is revealed a threaded hub that will fit into either trunnion as in Fig. 105 or into the special loop as in Figs. 95 and 106. The counterpoise plummet (C. P. P.) weighs 12 oz. and can be used for an extra plumb bob when not employed as above.

For mining and other surveys the adjustable plummet with inside reel as shown in Fig. 186 has been widely preferred and for surveys in windy country the slender mercury plummet has also had some popularity.

VALEDICTION

This publication has been prepared in the midst of allied responsibilities and does not profess to cover the whole argument under the various topics treated.

In the nature of things its very incompleteness must suggest grounds for debate which we shall gladly entertain in our effort to develop the intellectual aspect of business together with its commercial significance. Our civilzation has been developed by an intimate study of natural phenomena and our industries upon a susbstantial foundation of mutual confidence and co-operation. We acknowledge the exhilarating effect of our relations with the Engineering Proffession and take this occasion to pledge again our share of influnce with those manufacturers in America who are adding lustre to the national doctrine of honor and respectability in commerce and business relationship.

The greater achievements in the world's work are being accomplished with precision and dispatch by men who have been trained in courage, confidence and lofty ideals. It is to those who are so disposed that we are dedicating this little book.

We have not hesitated to adopt the select methods of collaborators who are trying to make work more agreeable by improving laboratory and field practice. We have studiously attempted to place credit where it belongs and dilligently sought to coördinate with precedent a frank expression of our views. The dominating impulse is an effort to meet the requirements in the best known way and to contribute the result of our own research toward the correction of fallacies, the removal of prejudice, the suppression of platitudes and the solution of our mutual problems on a scientific basis.

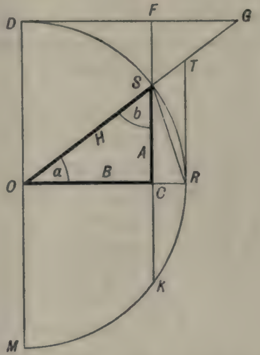
Our lectures are only cursory arguments in which the technicalities of each subject are utilized for the pardonable purpose of justifying the existence of our wares; but with comment and contribution from outside sources we hope ultimately to make this manual a clearing house for useful information on this subject.

APPENDIX

Trigonometrical Functions

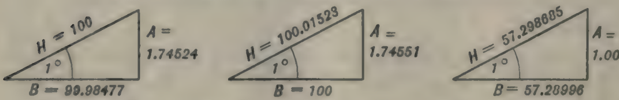
In the diagram let the angle SOC $= \mathbf{a}$, subtending chord SR and arc SR; let angle OSC $= \mathbf{b}$ and let the Altitude, Base and Hypothenuse be represented by the letters A, B and H. Then in any R. A. triangle, when OS = Rad. = 1,

Sine	**a**	= SC
Cosine	**a**	= OC
Tangent	**a**	= TR
Cotangent	**a**	= DG
Secant	**a**	= OT
Cosecant	**a**	= OG
Versine	**a**	= CR
Coversine	**a**	= SF
Exsecant	**a**	= ST
Coexsecant	**a**	= SG



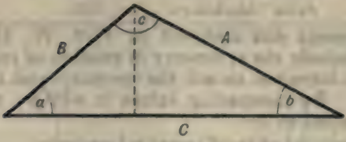
$$H = B \div \cos \mathbf{a} = A \div \sin \mathbf{a} = A \div \cos \mathbf{b} = B \div \sin \mathbf{b}$$
$$B = H \times \cos \mathbf{a} = A \times \cot \mathbf{a} = H \times \sin \mathbf{b} = A \times \tan \mathbf{b}$$
$$A = H \times \sin \mathbf{a} = B \times \tan \mathbf{a} = H \times \cos \mathbf{b} = B \times \cot \mathbf{b}$$
$$\sin \mathbf{a} = A \div H = \cos \mathbf{b} = \tan \mathbf{a} \times \cos \mathbf{a}$$
$$\cos \mathbf{a} = B \div H = \sin \mathbf{b} = 1 - \operatorname{vers} \mathbf{a}$$
$$\tan \mathbf{a} = A \div B = \cot \mathbf{b} = \sin \mathbf{a} \div \cos \mathbf{a}$$
$$\cot \mathbf{a} = B \div H = \tan \mathbf{b} = 1 \div \tan \mathbf{a}$$
$$\sec \mathbf{a} = H \div B = \operatorname{cosec} \mathbf{b} = 1 \div \cos \mathbf{a}$$
$$\operatorname{cosec} \mathbf{a} = H \div A = \sec \mathbf{b} = 1 \div \sin \mathbf{a}$$
$$\operatorname{vers} \mathbf{a} = (H - B) \div H = \operatorname{covers} \mathbf{b}$$
$$\operatorname{covers} \mathbf{a} = (H - A) \div H = \operatorname{vers} \mathbf{b}$$
$$\operatorname{exsec} \mathbf{a} = (H - B) \div B = \operatorname{coexsec} \mathbf{b}$$
$$\operatorname{coexsec} \mathbf{a} = (H - A) \div A = \operatorname{exsec} \mathbf{b}$$

Functions of a 1° Grade or 1° Random



In the third example the values of H and B are nearly identical with each other and with the numerical value of a radian. The angle which subtends an arc that is equal in length to the radius, is known as a radian. It is equal to $360° \div 2\pi = 57.29578$. This is therefore the numerical value of the radius of a circle in which one unit of length, measured along the circumference, subtends 1°.

Oblique Angle Triangles



Fundamental Formulae

$c = 180° - (a + b)$ etc.

$\sin a : \sin b :: A : B$

$\tan \frac{1}{2}(a + b) : \tan \frac{1}{2}(a - b) :: A + B : A - B$

Given	Sought	Formulae
a, c, A	C, B.	$C = A \div \sin a \sin c$; $B = A \div \sin a \sin (a + c)$
a, A, C	c, B.	$\sin c = C(\sin a \div A)$; $B = \sin b (A \div \sin a)$
a, c, A	C, B.	$C = \sin c (A \div \sin a)$; $B = \sin (a + c)(A \div \sin a)$
A, B, C.	a	Let $s = \frac{1}{2}(A + B + C)$, then
		$\sin a = \dfrac{2\sqrt{s(s-A)(s-B)(s-C)}}{C\ B}$
	area	$\sqrt{s(s-A)(s-B)(s-C)}$
a, b, c, A	area	$\dfrac{A^2 \sin c \sin b}{2 \sin a}$
b, A, C	area	$\frac{1}{2} A\ C \sin b$

Curve Location Formulae

R = Radius

D = Degree of curvature

L = Length of curve

Δ = Central angle

$R = T \cot \frac{1}{2}\Delta = E \div \operatorname{exsec} \frac{1}{2}\Delta$

$\Delta = 50 \div \sin \frac{1}{2}\Delta = 5730 \div D$

$L = 100\,\Delta \div D$

$\Delta = DL \div 100$

$D = 100\,\Delta \div L$

T = Tangent distance

C = Long Cord

M = Middle Ordinate

E = External distance

$T = R \tan \frac{1}{2}\Delta = E \cot \frac{1}{4}\Delta$

$C = 2 R \sin \frac{1}{2}\Delta$

$M = R \operatorname{vers} \frac{1}{2}\Delta$

$E = R \operatorname{exsec} \frac{1}{2}\Delta = T \tan \frac{1}{4}\Delta$

Random Correction

Given the length of one side and the offset, *sin* a may be found in correcting the direction of the random line by dividing the offset, A, by the measured distance, H; but where the correction angle is small and the lengths of the sides differ insensibly, the offset is equal to the arc. To find the deflection angle to turn from the random line, the **Radian Rule** may be applied as follows:

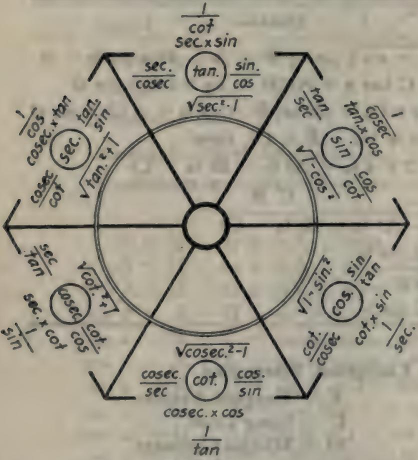
Multiply 57.3° by the offset and divide the product by the measured length of the random line. The result will be in decimals of degrees and will be a trifle large for, $a : 360° :: A : 2\pi r$.

Wilson's Functional Diagram

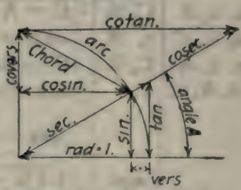
From "Machinery", Nov. 1903.

The arrangement was designed by Prof. W. H. Wilson in order to show concisely the symmetry of relation of trigonometrical functions. It is intended to aid the memory and to be used for ready reference. By comparing values it will be manifest that each function is equal to :—

The reciprocal of its diametrical opposite ;
The ratio of the first and second in either direction;
The product of its two adjacents, or
An expression involving the square of an adjacent, a radical and unity.

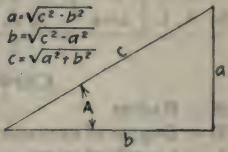


FUNCTIONS



MENSURATION

$a = \sqrt{c^2 - b^2}$
$b = \sqrt{c^2 - a^2}$
$c = \sqrt{a^2 + b^2}$



$\sin. A = \frac{a}{c}$	$a = c \sin A$	$c = \frac{a}{\sin. A}$	$\sin. A = \frac{1}{\text{cosec}. A}$
$\text{cosin}. A = \frac{b}{c}$	$b = c \text{ cosin}. A$	$c = \frac{b}{\text{cosin}. A}$	$\text{cosin}. A = \frac{1}{\sec. A}$
$\tan. A = \frac{a}{b}$	$a = b \tan. A$	$b = \frac{a}{\tan. A}$	$\tan. A = \frac{1}{\text{cotan}. A}$
$\text{cotan}. A = \frac{b}{a}$	$b = a \text{ cotan}. A$	$a = \frac{b}{\text{cotan}. A}$	$\text{cotan}. A = \frac{1}{\tan. A}$
$\sec. A = \frac{c}{b}$	$c = b \sec A$	$b = \frac{c}{\sec A}$	$\sec. A = \frac{1}{\text{cosin}. A}$
$\text{cosec}. A = \frac{c}{a}$	$c = a \text{ cosec}. A$	$a = \frac{c}{\text{cosec}. A}$	$\text{cosec} A = \frac{1}{\sin. A}$
$\text{vers}. A = \frac{c-b}{c}$	$c-b = c \text{ vers}. A$	$c = \frac{c-b}{\text{vers}. A}$	$\text{vers}. A = 1 - \text{cosin}. A$
$\text{covers}. A = \frac{c-a}{c}$	$c-a = c \text{ covers} A$	$c = \frac{c-a}{\text{covers}. A}$	$\text{covers} A = 1 - \sin. A$
$\text{suvers}. A = 1 + \frac{b}{c}$	$b = c(\text{suvers} A - 1)$	$c = \frac{b}{\text{suvers}. A - 1}$	$\text{suvers} A = 1 + \cos. A$

Required	Given	Formula
Periphery:		
Circle	Rad. = r	$2\pi r$
Side of equal Sq.	Diam. = D	.88623 D
Perimeter of equal Sq.	Rad. = r	$2\pi r \times 1.1284$
Side of Inscr. Sq.	Diam. = D	.7071 D
Area:		
Circle	Rad. = r; Diam = D	πr^2 or $.7854\ D^2$
Sector	Rad. = r; Arc = L	$\frac{1}{2} r L$
Segment	Rad. = r; Arc = L; Cent. angle = a	$\frac{1}{2} r (L - \sin a)$
Cone	Rad. of base = r; Slant height = s	πrs (add area of base)
Ellipse	Semi-axes = a and b	πab
Parabola	Cord = c; Height = h	$\frac{2}{3}$ ch
Trapezoid	Parallel sides = m and n; Altitude = p	$(m + n) \frac{p}{2}$
Reg. Polygon	Side = s; Number = n	$\frac{1}{4} s^2 n \times \cot \frac{180^\circ}{n}$
Pentagon	Side = s	$1.720477\ s^2$
Hexagon	Side = s	$2.598076\ s^2$
Octagon	Side = s	$4.828427\ s^2$
Inscribed Square	Radius = r	$0.63661\ \pi r^2$
Cicumscribed Square	Radius = r	$1.2732\ \pi r^2$
Inscribed Circle	Side of Square = s	$0.7854\ s^2$
Circumscribed Circle	Side of Square = s	$1.4142\ s$ = Diam.
Sphere	Radius = r, Diam. = d	$4\pi r^2$ or $\pi d^2 = 4$ G.C.
Sph. Segment	Height = h; Rad. = r_1	$\frac{h}{2} \pi r_1{}^2 + \frac{\pi h^3}{6}$
Sph. Zone	Rad. of Sph. = r; Height = h	$2\pi rh$
Cylinder	Radius = r; Height = h	$2\pi rh$
Volume:		
Prism or Cylinder	Base = b; Height = h	bh
Wedge	Base = b, Height = h	$\frac{1}{2}$ bh
Pyramid or Cone	Base = b; Height = h	$\frac{1}{3}$ bh
Frustum	Bases = b and b'	$\frac{1}{3} h (b + b' + \sqrt{bb'})$
Sphere	Radius = r	$\frac{4}{3} \pi r^3$ or $4.1888\ r^3$
Zone	Up. Rad. = r; Lower = r_1	$\frac{\pi h}{2} \left(r^2 + r_1{}^2 + \frac{h^2}{3}\right)$

The International Metric System

The essential features of this extraordinary system were embodied in a report issued by the French Acad. of Sci. in 1791. By 1840 it became generally adopted in France and since that time has been legalized or made compulsory in every civilized country throughout the world.

In 1864 it was permitted by the British Parliament, and in 1866 it was legalized by the American Congress. After 7 years deliberation, in 1868 an act was passed by the Reichstag making the system permissable at the beginning of 1870, and in spite of the Franco-Prussian War it was made compulsory in Germany, Jan. 1, 1872.

By 1870, in fact, the use of the meter as a basis of geodetic and cadastral surveys had become so general throughout Europe that a conference was called at Paris in that year for the purpose of organizing a central bureau where the standards of the various countries could be intercompared. As a result the International Bureau of Weights and Measures was permanently established by general consent on neutral ground at Breteuil, near Paris, in 1875 and the U. S. became one of the seventeen signatory nations at that time.

The great advantages recognized by all people engaged in international or interstate commerce include :—

1. The decimal relation between the units,
2. The extremely simple interrelation of the units of length, area, volume and weight, and
3. The uniform nomenclature and terminology.

This system constitutes one of the greatest single benefactions that has ever been contributed to international comity. The application of science to industry and life will ultimately make it the common standard of exchange between all people. Were it not for the inertia of Russia, England and the U. S. this would even now be an accomplished fact. So long as the individual States of America regulate the units of measurement, progress is certain to be tedious.

The Metric System has been widely used in every U. S. Govt. Dept. except the Rectilinear Survey under the General Land Office, where Gunter's Chain (66 ft.) has been used to the exclusion of every other unit of measure. Gunter's Chain is rapidly going into disuse, but where land is estimated in acres it is the best unit of measure because area, expressed in square chains, is immediately reduced to acres by moving the decimal point one place to the left.

We use the Metric System extensively in our optical, chemical and mechanical laboratories. To assist in popularizing the same, as well as to facilitate commerce with our friends in Mexico, Central and South America, we supply Tables of Equivalents compiled from reports of the U. S. Bureau of Standards and those extracted from the *American Civil Engineers' Pocket Book* with the compliments of Mansfield Merriman, Ed. in Chief.

Metric Nomenclature

Denomination	Abrev.	Multiple	Equivalent	
Myriameter	Mm	10,000 Meters	6.2137 Miles	
Kilometer	Km	1,000 "	.62137 "	
Hectometer	Hm	100 "	328 Ft. 1 In.	
Dekameter	Dm	10 "	10 Yds., 33.7 In.	
Meter (Unit)	m	(0.513074 Toises)	39.37 Inches*	By Act of Congress
Decimeter	dm	1/10 Meter	3.937 "	
Centimeter	cm	1/100 "	.3937 "	
Millimeter	mm	1/1000 "	.0394 "	
Micron (=.001 mm)	μ	1/1000000 "	1/2117 Line	

*The mean of five comparisons by Hassler, Kater, Bailey, Clark and Comstock is 39.36982.

Denomination	Abrev.	Multiple	Equivalent
Sq Kilometer	Km^2	1 Million Sq. Meters	0.386101 Sq. Miles
Hektar	Ha	10,000 " "	2.471 Acres
Dekar	Da	1,000 " "	.247 "
Ar (Unit)	a	100 " "	119.6 Sq. Yds
Centar (= m^2)	ca	1 " "	10.764 " Ft.
Sq. Decimeter	dm^2	1/100 " "	15.5 " Ins.
Sq. Centimeter	cm^2	1/10000 " "	.155 " "
Sq. Millimeter	mm^2	1/1000000 " "	.00155 " "

Denomination	Abrev.	Multiple	Dry	Liquid
Kiloliter or Ster	S	1000 l = 1 m^3	1.308 Cu Yds	264.17 U. S. Gal.
Hectoliter	Hl	100 l	2.8377 Bu	26.417 " "
Dekaliter	Dl	10 l	9.08 Qts; 1.135 Pk	2.642 " "
Liter (Unit)	l	1 dm^3	.908 Qts	1.0567 Qts
Deciliter	dl	100 cm^3	6.1023 Cu in.	0.846 Gills
Centiliter	cl	10 cm^3	.61023 "	0.3381 Fl. Oz.
Milliliter	ml	1 cm^3		0.27 Fl. Drm.
Cu. Millimeter	mm^3			
Microliter (=.001 ml)	λ			

Denomination	Abrev.	Grams	Of Water	Equivalent
Toneau	T	1,000,000.	1 m^3	2204.622 Lbs.
Quintal	Q	100,000.	1 Hl	.11023 Tons.
Myriagram	Mg	10,000.	10 l	22.04622 Lbs.
Kilogram	Kg	1,000.	1 l	2.2046 "
Hectogram	Hg	100.	1 dl	3.5274 Oz.
Dekagram	Dg	10.	10 cm^3	.3527 "
Gram (Unit)	g	1.	1 cm^3	15.4324 Grains
Decigram	dg	.1		.0772 Scruple
Centigram	cg	.01		.0026 Dram
Milligram	mg	.001	1 mm^3	.015432 Grain
Microgram (=.001 mg)	γ			

NOTE: Metric Charts are published by the Bureau of Standards, Washington, D. C., and are to be had on application, to S. W. Stratton, Director.

From a Report of the U. S. Bureau of Standards

LENGTH

Inches.	Milli-meters.	Inches.	Centi-meters.	Feet.	Meters.	U. S. yards.	Meters.	U. S. miles.	Kilo-meters.
0.03937 =	1	0.3937 =	1	1 =	0.304801	1 =	0.914402	0.62137 =	1
0.07874 =	2	0.7874 =	2	2 =	0.609601	1.093611 =	1	1 =	1.60935
0.11811 =	3	1 =	2.54001	3 =	0.914402	2 =	1.828804	1.24274 =	2
0.15748 =	4	1.1811 =	3	3.28083 =	1	2.187222 =	2	1.86411 =	3
0.19685 =	5	1.5748 =	4	4 =	1.219202	3 =	2.743205	2 =	3.21869
0.23622 =	6	1.9685 =	5	5 =	1.524003	3.280833 =	3	2.48548 =	4
0.27559 =	7	2 =	5.08001	6 =	1.828804	4 =	3.657607	3 =	4.82804
0.31496 =	8	2.3622 =	6	6.56167 =	2	4.374444 =	4	3.10685 =	5
0.35433 =	9	2.7559 =	7	7 =	2.133604	5 =	4.572009	3.72822 =	6
1 =	25.4001	3 =	7.62002	8 =	2.438405	5.468056 =	5	4 =	6.43739
2 =	50.8001	3.1496 =	8	9 =	2.743205	6 =	5.486411	4.34959 =	7
3 =	76.2002	3.5433 =	9	9.84250 =	3	6.561667 =	6	4.97096 =	8
4 =	101.6002	4 =	10.16002	13.12333 =	4	7 =	6.400813	5 =	8.04674
5 =	127.0003	5 =	12.70003	16.40417 =	5	7.655278 =	7	5.59233 =	9
6 =	152.4003	6 =	15.24003	19.68500 =	6	8 =	7.315215	6 =	9.65608
7 =	177.8004	7 =	17.78004	22.96583 =	7	8.748889 =	8	7 =	11.26543
8 =	203.2004	8 =	20.32004	26.24667 =	8	9 =	8.229616	8 =	12.87478
9 =	228.6005	9 =	22.86005	29.52750 =	9	9.842500 =	9	9 =	14.48412

AREA.

Square inches.	Square milli-meters.	Square inches.	Square centi-meters.	Square feet.	Square meters.	Square yards.	Square meters.	Square miles.	Square kilo-meters.
0.00155 =	1	0.1550 =	1	1 =	0.09290	1 =	0.8361	0.3861 =	1
0.00310 =	2	0.3100 =	2	2 =	0.18581	1.1960 =	1	0.7722 =	2
0.00465 =	3	0.4650 =	3	3 =	0.27871	2 =	1.6723	1 =	2.5900
0.00620 =	4	0.6200 =	4	4 =	0.37161	2.3920 =	2	1.1583 =	3
0.00775 =	5	0.7750 =	5	5 =	0.46452	3 =	2.5084	1.5444 =	4
0.00930 =	6	0.9300 =	6	6 =	0.55742	3.5880 =	3	1.9305 =	5
0.01085 =	7	1 =	6.452	7 =	0.65032	4 =	3.3445	2 =	5.1800
0.01240 =	8	1.0850 =	7	8 =	0.74323	4.7839 =	4	2.3166 =	6
0.01395 =	9	1.2400 =	8	9 =	0.83613	5 =	4.1807	2.7027 =	7
1 =	645.16	1.3950 =	9	10.764 =	1	5.9799 =	5	3 =	7.7700
2 =	1,290.33	2 =	12.903	21.528 =	2	6 =	5.0168	3.0888 =	8
3 =	1,935.49	3 =	19.355	32.292 =	3	7 =	5.8529	3.4749 =	9
4 =	2,580.65	4 =	25.807	43.055 =	4	7.1759 =	6	4 =	10.3600
5 =	3,225.81	5 =	32.258	53.819 =	5	8 =	6.6890	5 =	12.9500
6 =	3,870.98	6 =	38.710	64.583 —	6	8.3719 =	7	6 =	15.5400
7 =	4,516.14	7 =	45.161	75.347 =	7	9 =	7.5252	7 =	18.1300
8 =	5,161.30	8 =	51.613	86.111 =	8	9.5679 =	8	8 =	20.7200
9 =	5,806.46	9 =	58.065	96.875 =	9	10.7639 =	9	9 =	23.3100

VOLUME. / AREA—cont'd.

Cubic inches.	Cubic millimeters.	Cubic inches.	Cubic centimeters.	Cubic feet	Cubic meters.	Cubic yards.	Cubic meters.	Acres.	Hectares.
0.000061 =	1	0.0610 =	1	1 =	0.02832	1 =	0.7646	1 =	0.4047
0.000122 =	2	0.1220 =	2	2 =	0.05663	1.3079 =	1	2 =	0.8094
0.000183 =	3	0.1831 =	3	3 =	0.08495	2 =	1.5291	2.471 =	1
0.000244 =	4	0.2441 =	4	4 =	0.11327	2.6159 =	2	3 =	1.2141
0.000305 =	5	0.3051 =	5	5 =	0.14159	3 =	2.2937	4 =	1.6187
0.000366 =	6	0.3661 =	6	6 =	0.16990	3.9238 =	3	4.942 =	2
0.000427 =	7	0.4272 =	7	7 =	0.19822	4 =	3.0582	5 =	2.0234
0.000488 =	8	0.4882 =	8	8 =	0.22654	5 =	3.8228	6 =	2.4281
0.000549 =	9	0.5492 =	9	9 =	0.25485	5.2318 =	4	7 =	2.8328
1 =	16,387.2	1 =	16.3872	35.314 =	1	6 =	4.5874	7.413 =	3
2 =	32,774.3	2 =	32.7743	70.629 =	2	6.5397 =	5	8 =	3.2375
3 =	49,161.5	3 =	49.1615	105.943 =	3	7 =	5.3519	9 =	3.6422
4 =	65,548.6	4 =	65.5486	141.258 =	4	7.8477 =	6	9.884 =	4
5 =	81,935.8	5 =	81.9358	176.572 =	5	8 =	6.1165	12.355 =	5
6 =	98,323.0	6 =	98.3230	211.887 =	6	9 =	6.8810	14.826 =	6
7 =	114,710.1	7 =	114.7101	247.201 =	7	9.1556 =	7	17.297 =	7
8 =	131,097.3	8 =	131.0973	282.516 =	8	10.4635 =	8	19.768 =	8
9 =	147,484.5	9 =	147.4845	317.830 =	9	11.7715 =	9	22.239 =	9

From a Report of the U. S. Bureau of Standards

CAPACITY.

Milliliters. (cc.)	U. S. liquid ounces.	Milliliters. (cc.)	U. S. apothecaries' drams.	U. S. apothecaries' scruples.	Milliliters. (cc.)	U. S. liquid quarts.	Liters.	U. S. liquid gallons.	Liters.
1	0.03381	1	0.2705	0.8115	1	1	0.94636	0.26417	1
2	0.06763	2	0.5410	1	1.2322	1.05668	1	0.52834	2
3	0.10144	3	0.8115	1.6231	2	2	1.89272	0.79251	3
4	0.13526	3.6967	1	2	2.4645	2.11336	2	1	3.78543
5	0.16907	4	1.0820	2.4346	3	3	2.83908	1.05668	4
6	0.20288	5	1.3525	3	3.6967	3.17005	3	1.32085	5
7	0.23670	6	1.6231	3.2461	4	4	3.78543	1.58502	6
8	0.27051	7	1.8936	4	4.9290	4.22673	4	1.84919	7
9	0.30432	7.3934	2	4.0577	5	5	4.73179	2	7.57087
29.574	1	8	2.1641	4.8692	6	5.28341	5	2.11336	8
59.147	2	9	2.4346	5	6.1612	6	5.67815	2.37753	9
88.721	3	11.0901	3	5.6807	7	6.34009	6	3	11.35630
118.295	4	14.7869	4	6	7.3934	7	6.62451	4	15.14174
147.869	5	18.4836	5	6.4923	8	7.39677	7	5	18.92717
177.442	6	22.1803	6	7	8.6257	8	7.57088	6	22.71261
207.016	7	25.8770	7	7.3038	9	8.45345	8	7	26.49804
236.590	8	29.5737	8	8	9.8579	9	8.51723	8	30.28348
266.163	9	33.2704	9	9	11.0901	9.51014	9	9	34.06891

U. S. dry quarts.	Liters.	U. S. pecks.	Liters.	Dekaliters.	U. S. pecks.	U. S. bushels.	Hectoliters.	U. S. bushels per acre.	Hectoliters per hectare.
0.9081	1	0.11351	1	0.8810	1	1	0.35239	1	0.87078
1	1.1012	0.22702	2	1	1.1351	2	0.70479	1.14840	1
1.8162	2	0.34053	3	1.7620	2	2.83774	1	2	1.74156
2	2.2025	0.45404	4	2	2.2702	3	1.05718	2.29680	2
2.7242	3	0.56755	5	2.6429	3	4	1.40957	3	2.61233
3	3.3037	0.68106	6	3	3.4053	5	1.76196	3.44519	3
3.6323	4	0.79457	7	3.5239	4	5.67548	2	4	3.48311
4	4.4049	0.90808	8	4	4.5404	6	2.11436	4.59359	4
4.5404	5	1	8.80982	4.4049	5	7	2.46675	5	4.35389
5	5.5061	1.02157	9	5	5.6755	8	2.81914	5.74199	5
5.4485	6	2	17.61964	5.2859	6	8.51323	3	6	5.22467
6	6.6074	3	26.42946	6	6.8106	9	3.17154	6.89039	6
6.3565	7	4	35.23928	6.1669	7	11.35097	4	7	6.09545
7	7.7086	5	44.04910	7	7.9457	14.18871	5	8	6.96622
7.2646	8	6	52.85892	7.0479	8	17.02645	6	8.03879	7
8	8.8098	7	61.66874	7.9288	9	19.86420	7	9	7.83700
8.1727	9	8	70.47856	8	9.0808	22.70194	8	9.18719	8
9	9.9110	9	79.28838	9	10.2159	25.53968	9	10.33558	9

WEIGHT (OR MASS).

Grains.	Grams.	Avoirdupois ounces.	Grams.	Troy ounces.	Grams.	Avoirdupois pounds.	Kilograms.	Troy pounds.	Kilograms.
1	0.06480	0.03527	1	0.03215	1	1	0.45359	1	0.37324
2	0.12960	0.07055	2	0.06430	2	2	0.90718	2	0.74648
3	0.19440	0.10582	3	0.09645	3	2.20462	1	2.67923	1
4	0.25920	0.14110	4	0.12860	4	3	1.36078	3	1.11973
5	0.32399	0.17637	5	0.16075	5	4	1.81437	4	1.49297
6	0.38879	0.21164	6	0.19290	6	4.40924	2	5	1.86621
7	0.45359	0.24692	7	0.22506	7	5	2.26796	5.35846	2
8	0.51839	0.28219	8	0.25721	8	6	2.72155	6	2.23945
9	0.58319	0.31747	9	0.28936	9	6.61387	3	7	2.61269
15.4324	1	1	28.3495	1	31.10348	7	3.17515	8	2.98593
30.8647	2	2	56.6991	2	62.20696	8	3.62874	8.03769	3
46.2971	3	3	85.0486	3	93.31044	8.81849	4	9	3.35918
61.7294	4	4	113.3981	4	124.41392	9	4.08233	10.71691	4
77.1618	5	5	141.7476	5	155.51740	11.02311	5	13.39614	5
92.5941	6	6	170.0972	6	186.62088	13.22773	6	16.07537	6
108.0265	7	7	198.4467	7	217.72437	15.43236	7	18.75460	7
123.4589	8	8	226.7962	8	248.82785	17.63698	8	21.43383	8
138.8912	9	9	255.1457	9	279.93133	19.84160	9	24.11306	9

Length

1 meter = 10 decimeters = 100 centimeters = 1000 millimeters = 10^6 microns = 0.1 dekameter = 0.01 hectometer = 0.001 kilometer = 0.0001 myriameter.

1 U. S. yard = 3600/3937 meters (by definition); log = $\bar{1}$.9611371.

Meters	Inches	Feet	Yards	Links	Rods, poles, or perches	Chains, Gunter's	Statute miles U. S.	Nautical miles U. S.
1	39.37 1.59517*	3.2808 0.51598*	1.0936 0.03886*	4.971 0.69644*	0.1988 $\bar{1}$.29850*	0.04971 $\bar{2}$.69644*	0.0$_3$6214 $\bar{4}$.79335*	0.0$_3$5396 $\bar{4}$.73207*
0.0254 $\bar{2}$.40483*	1	0.08333 $\bar{2}$.92082*	0.02778 $\bar{2}$.44370*	0.1263 $\bar{1}$.10127*	0.005051 $\bar{3}$.70333*	0.001263 $\bar{3}$.10127*	0.0$_4$1578 $\bar{5}$.19818*	0.0$_4$1371 $\bar{5}$.13690
0.3048 $\bar{1}$.48402*	12. 1.07918*	1	0.3333 $\bar{1}$.52288*	1.515 0.18046*	0.06061 $\bar{2}$.78252*	0.01515 $\bar{2}$.18046*	0.0$_3$1894 $\bar{4}$.27737*	0.0$_3$1645 $\bar{4}$.21608*
0.9144 $\bar{1}$.96114*	36. 1.55630*	3. 0.47712*	1	4.545 0.65758*	0.1818 $\bar{1}$.25964*	0.04545 $\bar{2}$.65758*	0.0$_3$5682 $\bar{4}$.75449*	0.0$_3$4934 $\bar{4}$.69320*
0.2012 $\bar{1}$.30356*	7.92 0.89873*	0.66 $\bar{1}$.81954*	0.22 $\bar{1}$.34242*	1	0.04 $\bar{2}$.60206*	0.01 $\bar{2}$.00000*	0.0$_3$1250 $\bar{4}$.09691*	0.0$_3$1086 $\bar{4}$.03564*
5.029 0.70150*	198. 2.29667*	16.5 1.21748*	5.5 0.74036*	25. 1.39794*	1	0.25 $\bar{1}$.39794*	0.0$_2$3125 $\bar{3}$.49485*	0.0$_2$2714 $\bar{3}$.43357*
20.12 1.30356*	792. 2.89873*	66. 1.81954*	22. 1.34242*	100. 2.00000*	4. 0.60206*	1	0.0125 $\bar{2}$.09691*	0.01086 $\bar{2}$.03564*
1609.3 3.20665*	63360. 4.80182*	5280. 3.72263*	1760. 3.24551*	8000. 3.90309*	320. 2.50515*	80. 1.90309*	1	0.8684 $\bar{1}$.93872
1853.25 3.26793*	72962 4.86310*	6080.2 3.78392*	2026.73 3.30680*	9212 3.96437*	368.5 2.56643*	92.12 1.96437*	1.1516 0.06128*	1

1 nautical mile of the British admiralty = 6080 ft. 1 furlong = ⅛ mile = 660 feet. 1 league = 3 miles = 24 furlongs. 1 fathom = 2 yards = 6 feet.

* Logarithm of the number immediately above.

Area

1 hectare = 100 ares = 10 000 centares or square meters.

Square meters	Square inches	Square feet	Square yards	Square rods	Square chains	Acres	Square miles or sections
1	1550 3.19033*	10.764 1.03197*	1.1960 0.07773*	0.03954 $\bar{2}$.59700*	0.0$_2$2471 $\bar{3}$.39288*	0.0$_3$2471 $\bar{4}$.39288*	0.0$_6$3861 $\bar{7}$.58670*
0.0$_3$6452 $\bar{4}$.80967*	1	0.0$_2$6944 $\bar{3}$.84164*	0.0$_3$7716 $\bar{4}$.88740*	0.0$_4$2551 $\bar{5}$.40667*	0.0$_5$1594 $\bar{6}$.20255*	0.0$_6$1594 $\bar{7}$.20255*	0.0$_9$2491 $\bar{10}$.39637*
0.09290 $\bar{2}$.96803*	144 2.15836*	1	0.1111 $\bar{1}$.04576*	0.0$_2$3673 $\bar{3}$.56503*	0.0$_3$2296 $\bar{4}$.36091*	0.0$_4$2296 $\bar{5}$.36091*	0.0$_7$3587 $\bar{8}$.55473*
0.8361 $\bar{1}$.92227*	1296 3.11260*	9 0.95424*	1	0.03306 $\bar{2}$.51927*	0.0$_2$2066 $\bar{3}$.31515*	0.0$_3$2066 $\bar{4}$.31515*	0.0$_6$3228 $\bar{7}$.50898*
25.29 1.40300*	39204 4.59333*	272.25 2.43497*	30.25 1.48072*	1	0.0625 $\bar{2}$.79588*	0.00625 $\bar{3}$.79588*	0.0$_5$9766 $\bar{6}$.98970*
404.69 2.60712*	627264 5.79745*	4356 3.63909*	484 2.68484*	16 1.20412*	1	0.1 $\bar{1}$.00000*	0.0$_3$1562 $\bar{4}$.19382*
4046.9 3.60712*	6272640 6.79745*	43560 4.63909*	4840 3.68484*	160 2.20412*	10 1.00000*	1	0.001562 $\bar{3}$.19382*
2589998 6.41330*		27878400 7.44527*	3097600 6.49102*	102400 5.01030*	6400 3.80618*	640 2.80618*	1

* Logarithm of the number immediately above.

Speed and Velocity

Cm per sec	Km per hour	Ft per sec	Ft per min	Miles per hour	Knots
1	0.036 $\bar{2}$.55630*	0.03281 $\bar{2}$.51598*	1.9685 0.29413*	0.02237 $\bar{2}$.34965*	0.01942 $\bar{2}$.28825*
27.7778 1.44370*	1	0.91134 $\bar{1}$.95968*	54.6806 1.73783*	0.62137 $\bar{1}$.79335*	0.53960 $\bar{1}$.73207*
30.4801 1.48402*	1.0973 0.04032*	1	60 1.77815*	0.68182 $\bar{1}$.83367*	0.59209 $\bar{1}$.77238*
0.5080 $\bar{1}$.70586*	0.01829 $\bar{2}$.26217*	0.01667 $\bar{2}$.22185*	1	0.01136 $\bar{2}$.05553*	0.009868 $\bar{3}$.99423*
44.7041 1.65035*	1.6093 0.20670*	1.46667 0.16633*	88 1.94448*	1	0.86839 $\bar{1}$.93872*
51.4971 1.71178*	1.8532 0.26793*	1.68894 0.22761*	101.337 2.00577*	1.15155 0.06128*	1

1 knot = 1 nautical mile per hour.
* Logarithm of the number immediately above.

Volume and Capacity

1 liter = 1 cubic decimeter = 1000 cubic centimeters = 10 deciliters = 100 centiliters = 1000 milliliters = 0.1 dekaliter = 0.01 hectoliter = 0.01 kiloliter = 0.001 cubic meters or steres.

Cubic inches	Cubic feet	Cubic yards	U. S. quarts		Gallons		Bushels U. S.	Liters
			Liquid	Dry	U. S. liquid	U. S. dry		
1	0.0$_3$57870 $\bar{4}$.76246*	0.0$_4$2143 $\bar{5}$.33109*	0.017316 $\bar{2}$.23845*	0.014881 $\bar{2}$.17263*	0.004329 $\bar{3}$.63639*	0.003720 $\bar{3}$.57057*	0.0$_3$4650 $\bar{4}$.66748*	0.016387 $\bar{2}$.21450*
1728. 3.23754*	1	0.037037 $\bar{2}$.56864*	29.922 1.47599*	25.714 1.41017*	7.4805 0.87393*	6.4285 0.80811*	0.80356 $\bar{1}$.90502*	28.317 1.45205*
46656. 4.66891*	27. 1.43136*	1	807.90 2.90736*	694.28 2.84153*	201.97 2.30530*	173.57 2.23948*	21.696 1.33638*	764.56 2.88341*
57.75 1.76155*	0.033420 $\bar{2}$.52401*	0.001238 $\bar{3}$.09026*	1	0.85937 $\bar{1}$.93418*	0.25 $\bar{1}$.39794*	0.21484 $\bar{1}$.33212*	0.026855 $\bar{2}$.42903*	0.94636 $\bar{1}$.97606*
67.201 1.82737*	0.038889 $\bar{2}$.58983*	0.001440 $\bar{3}$.15847*	1.1637 0.06582*	1	0.29091 $\bar{1}$.46376*	0.25 $\bar{1}$.39794*	0.03125 $\bar{2}$.49485*	1.1012 0.04188*
231. 2.36361*	0.13368 $\bar{1}$.12607*	0.004951 $\bar{3}$.69471*	4. 0.60206*	3.4375 0.53624*	1	0.85937 $\bar{1}$.93418*	0.10742 $\bar{1}$.03109*	3.7854 0.57812*
268.80 2.42943*	0.15556 $\bar{1}$.19189*	0.005761 $\bar{3}$.76053*	4.6546 0.66788*	4. 0.60201*	1.1637 0.06582*	1	0.125 $\bar{1}$.09691*	4.4049 0.64394*
2150.4 3.33253*	1.2445 0.09498*	0.046091 $\bar{2}$.66362*	37.237 1.57097*	32. 1.50515*	9.3092 0.96891*	8 0.90309*	1	35.239 1.54703*
61.023 1.78550*	0.035315 $\bar{2}$.54795*	0.001308 $\bar{3}$.11659*	1.0567 0.02394*	0.90808 $\bar{1}$.95812*	0.26417 $\bar{1}$.42188*	0.22702 $\bar{1}$.35606*	0.028377 $\bar{2}$.45297*	1

1 U. S. liquid quart = 2 pints = 8 gills = 32 fluid ounces = 256 fluid drams = 768 fluid scruples. 1 bushel = 4 pecks.

1 Imperial gallon = 1.201 U. S. gallons = 0.1605 cu ft = 4.5460 liters.

1 U. S. gallon = 0.8327 Imperial gallons. 1 cubic foot = 6.229 Imperial gallons.

1 British bushel = 1.2837 cubic feet.

Shipping Measure: 1 register ton = 100 cu ft. 1 U. S. shipping ton = 40 cu ft.
1 British shipping ton = 42 cu ft

* Logarithm of the number immediately above.

Mass (Physicists' system) or **Force** (Engineers' system)

1 kilogram = 1000 grams = 0.001 metric ton. 1 gm = 10 decigrams = 100 centigrams = 1000 milligrams = 0.1 dekagram = 0.01 hectogram = 0.001 kilogram = 0.0001 myriagram.
1 U. S. Avoirdupois pound = 0.4535924277 kg = (by definition) 7000/5760 troy pounds.

Kilograms	Grains	Ounces		Pounds		Tons		
		Avoir.	Troy and apoth.	Troy and apoth.	Avoir.	Short, 2000 lb	Long, 2240 lb	Metric, 1000 kg
1	15432. 4.18843*	35.274 1.54745*	32.151 1.50719*	2.6792 0.42801*	2.20461 0.34333*	0.001102 $\bar{3}$.04230*	0.0_39842 $\bar{4}$.99309*	**0.001** $\bar{3}$.00000*
0.0_46480 $\bar{5}$.81157*	1	0.0_22286 $\bar{3}$.35902*	0.002083 $\bar{3}$.31876*	0.0_31736 $\bar{4}$.23958*	0.0_3142 $\bar{4}$.15490*			
0.028349 $\bar{2}$.45255*	**437.5** 2.64098*	1	0.91146 $\bar{1}$.95974*	0.075955 $\bar{2}$.88056*	**0.0625** $\bar{2}$.79588*	**0.0_43125** $\bar{5}$.49485*	0.0_42790 $\bar{5}$.44563*	0.0_42835 $\bar{5}$.45255*
0.031103 $\bar{2}$.49281*	**480.** 2.68124*	1.0971 0.04026*	1	0.083333 $\bar{2}$.92082*	0.068571 $\bar{2}$.83614*	0.0_43429 $\bar{5}$.53511*	0.0_43061 $\bar{4}$.56508*	0.0_43110 $\bar{5}$.49281*
0.37324 $\bar{1}$.57199*	**5760.** 3.76042*	13.166 1.11944*	**12.** 1.07918*	1		0.0_34114 $\bar{4}$.61429*	0.0_33673 $\bar{4}$.56508*	0.0_33732 $\bar{4}$.57199*
0.45359 $\bar{1}$.65667*	**7000.** 3.84510*	**16.** 1.20412*	14.583 1.16386*	1.2153 0.08468*	1	**0.0005** $\bar{4}$.69897*	0.0_34464 $\bar{4}$.64975*	0.0_34536 $\bar{4}$.65667*
907.18 2.95770*		**32000.** 4.50515*	29167. 4.46489*	2430.6 3.38570*	**2000.** 3.30103*	1	0.89286 $\bar{1}$.95078*	0.90718 $\bar{1}$.95770*
1016.1 3.00691*		**35840.** 4.55437*	32667 4.51410*	2722.2 3.43492*	**2240.** 3.35025*	**1.12** 0.04922*	1	1.0160 0.00691*
1000. 3.00000*		35274 4.54745*	32151 4.50719*	2679.2 3.42801*	2204.6 3.34333*	1.1023 0.04230*	0.98421 $\bar{1}$.99309*	1

1 quarter = 28 lb avoir. 1 pennyweight = 24 gr = 0.05 oz troy. 1 oz avoir. = 16 drams avoir. = 437.5 gr. 1 stone = 14 pounds. 1 cental = 100 pounds. 1 hundredweight = 112 pounds. 1 apothecaries' ounce = 8 apoth. drams = 24 scruples = 480 grains.

* Logarithm of the number immediately above.

Power

1 kilowatt = 1000 watts = 1000 joules per second.
1 horse-power = 550 foot-pounds per second.
1 cheval-vapeur = 75 kilogram-meters per second.

Kilowatts	Cheval-vapeur	Poncelet	Horse-power	M-kg per sec	Ft-lb per sec	Kg cal per sec	Btu per sec
1	1.3600 0.13341*	1.0197 0.00848*	1.341 0.12743*	101.97 2.00848*	737.5 2.86780*	0.2388 $\bar{1}$.37803*	0.9475 $\bar{1}$.97660*
0.7355 $\bar{1}$.86659*	1	**0.75** $\bar{1}$.87506*	0.9863 $\bar{1}$.99402*	**75** 1.87506*	542.5 2.73438*	0.1756 $\bar{1}$.24456*	0.6969 $\bar{1}$.84318
0.980665 $\bar{1}$.99152*	1.333 0.12493*	1	1.3151 0.11896*	**100** 2.00000*	723.3 2.85932*	0.2342 $\bar{1}$.36951*	0.9292 $\bar{1}$.96812*
0.7457 $\bar{1}$.87257*	1.0139 0.00598*	0.7604 $\bar{1}$.88104*	1	76.04 1.88104*	**550** 2.74036*	0.1780 $\bar{1}$.25055*	0.7066 $\bar{1}$.84916*
0.009807 $\bar{3}$.99152*	0.01333 $\bar{2}$.12493*	**0.01** $\bar{2}$.00000*	0.01315 $\bar{2}$.11896*	1	7.233 0.85932*	0.002342 $\bar{3}$.36951*	0.009292 $\bar{3}$.96812*
0.001356 $\bar{3}$.13220*	0.001843 $\bar{3}$.26562*	0.00138 $\bar{3}$.14068*	0.001818 $\bar{3}$.25964*	0.1383 $\bar{1}$.14068*	1	0.0003237 $\bar{4}$.51016*	0.001285 $\bar{3}$.10880*
4.188 0.62201*	5.694 0.75542*	4.271 0.63049*	5.616 0.74945*	427.1 2.63049*	3089 3.48984*	1	3.968 0.59861*
1.055 0.02340*	1.435 0.15682*	1.076 0.03188*	1.415 0.15084*	107.62 2.03188*	778.4 2.89120*	0.2520 $\bar{1}$.40139*	1

* Logarithm of the number immediately above.

Pressure

Kilograms per sq cm	Pounds		Short tons, per sq ft	Atmospheres	Columns of mercury†		Columns of water†	
	Per sq in	Per sq ft			Meters	Inches	Meters	Feet
1	14.223 1.15300*	2048.2 3.31137*	1.0241 0.01034*	0.96781 $\bar{1}.98579$*	0.73553 $\bar{1}.86660$*	28.958 1.46177*	10.009 1.00038*	32.837 1.51636*
0.070307 $\bar{2}.84700$*	1	144. 2.15836*	0.072 $\bar{2}.85733$*	0.06804 $\bar{2}.83279$*	0.051713 $\bar{2}.71360$*	2.0359 0.30876*	0.70368 $\bar{1}.84738$*	2.3087 0.36336*
$0.0_{3}4882$ $\bar{4}.68863$*	0.006944 $\bar{3}.84164$*	1	0.0005 $\bar{4}.69897$*	007243.5 $\bar{4}.67442$*	$0.0_{3}3591$ $\bar{4}.55524$*	0.014138 $\bar{2}.15040$*	0.004887 $\bar{3}.68901$*	0.016032 $\bar{2}.20500$*
0.97648 $\bar{1}.98966$*	13.889 1.14267*	2000. 3.30103*	1	0.94504 $\bar{1}.97545$*	0.71823 $\bar{1}.85627$*	28.277 1.45143*	9.7734 0.99004*	32.065 1.50603*
1.0333 0.01421*	14.697 1.16722*	2116.3 3.32558*	1.0582 0.02955*	1	0.76 $\bar{1}.88081$*	29.921 1.47598*	10.342 1.01459*	33.929 1.53058*
1.3596 0.13340*	19.338 1.28640*	2784.6 3.44476*	1.3923 0.14373*	1.3158 0.11919*	1	39.37 1.59517*	13.607 1.13378*	44.644 1.64976*
0.034533 $\bar{2}.53323$*	0.49118 $\bar{1}.69124$*	70.729 1.84960*	0.035365 $\bar{2}.54857$*	0.033421 $\bar{2}.52402$*	0.025400 $\bar{2}.40484$*	1	0.34563 $\bar{1}.53861$*	1.1340 0.05460*
0.099913 $\bar{2}.99962$*	1.4211 0.15262*	204.64 2.31099*	0.10232 $\bar{1}.00996$*	0.096697 $\bar{2}.98541$*	0.073489 $\bar{2}.86622$*	2.8933 0.46139*	1	3.2808 0.51598*
0.030453 $\bar{2}.48364$*	0.43315 $\bar{1}.63664$*	62.374 1.79500*	0.031187 $\bar{2}.49397$*	0.029473 $\bar{2}.46942$*	0.022399 $\bar{2}.35024$*	0.88187 $\bar{1}.94540$*	0.30480 $\bar{1}.48402$*	1

* Logarithm of the number immediately above. † At 15° C. and $g = g_0$.

Energy

Joules $= 10^7$ erg	Meter-kilograms	Foot-pounds	Kilowatt-hours	Cheval-vapeur-hours	Horse-power-hours	British thermal units
1	0.10197 $\bar{1}.00848$*	0.73756 $\bar{1}.86780$*	$0.0_{6}27778$ $\bar{7}.44370$*	$0.0_{6}37767$ $\bar{7}.57711$*	$0.0_{6}37251$ $\bar{7}.57113$*	$0.0_{3}9475$ $\bar{4}.97660$*
9.80665 0.9915207*	1	7.2330 0.85932*	$0.0_{5}27241$ $\bar{6}.43522$*	$0.0_{5}37037$ $\bar{6}.56863$*	$0.0_{5}36530$ $\bar{6}.56265$*	0.009292 $\bar{3}.96812$*
1.3558 0.13220*	0.13826 $\bar{1}.14068$*	1	$0.0_{6}37662$ $\bar{7}.57590$*	$0.0_{6}51206$ $\bar{7}.70932$*	$0.0_{6}50505$ $\bar{7}.70333$*	0.001285 $\bar{3}.10880$*
3.6×10^6 6.55630*	3.6710×10^5 5.56478*	2.6552×10^6 6.42410*	1	1.3596 0.13342*	1.3410 0.12743*	3411. 3.53290*
2.6478×10^6 6.42288*	270000. 5.43136*	1.9529×10^6 6.29068*	0.73550 $\bar{1}.86658$*	1	0.98631 $\bar{1}.99401$*	2509. 3.39948*
2.6845×10^6 6.42887*	2.7375×10^5 5.43735*	1.98×10^6 6.29667*	0.74571 $\bar{1}.87257$*	1.0139 0.00598*	1	2544. 3.40547*
1055. 3.02340*	107.6 2.03188*	778.4 2.89120*	$0.0_{3}2932$ $\bar{4}.46710$*	$0.0_{3}3986$ $\bar{4}.60051$*	$0.0_{3}3931$ $\bar{4}.59453$*	1

* Logarithm of the number immediately above.

English Linear Measure

Inches	Feet	Yards	Rods	Furlongs	Miles
1.	0.083	0.028	0.00505	0.00012626	0.0000157828
12.	1.	0.333	0.06060	0.00151515	0.00018939
36.	3.	1.	0.1818	0.004545	0.00056818
198.	16.5	5.5	1.	0.025	0.003125
7920.	660.	220.	40.	1.	0.125
63360.	5280.	1760.	320.	8.	1.

Fanega	Spain	1.587	acres
Feddan	Egypt	1.03	acres
Frail (= ½ Barrel)	Spain	50.	lbs.
Frasco	Argentine Rep.	2.51	qts.
	Mexico	2.50	qts.
Frasila	Zanzibar	35.	lbs.
Fuder	Luxemburg	264.17	gal.
Funt = (96 zolotniks)	Russia	.903	lbs.
Gallon (Imp'l.)	Canada	277.	cu. in.
Garnez	Russian Poland	.865	gal.
Joch	Austria	1.422	acres
Ken	Japan	5.965	ft.
Koku	Japan	4.963	bu.
Korree	Russia	3.50	bu.
Kwan (= 1000 mommes)	Japan	8.267	lbs.
Last	Belg. and Holland	85.134	bu.
	England	85.520	bu.
	Prussia	112.290	bu.
	Russian Poland	11.375	bu.
	Spain	4760.	lbs.
League	Nautical	3.	mi.
	French (ancien)	2.764	mi.
	Paraguay	4033.	acres
Legua	Spain and Cal.	2.634	mi.
	China	.0126	in.
Libra	Argentine Rep.	1.013	lbs.
	Central America	1.043	lbs.
	Chili, Peru and Ur.	1.014	lbs.
	Cuba and Venez.	1.016	lbs.
	Mexico	1.015	lbs.
	Portugal	1.011	lbs.
	Spain	1.014	lbs.
Ligne	French (usuel)	.091	in.
Livre	French (usuel)	1.103	lbs.
	Greece	1.103	lbs.
	Guiana	1.079	lbs.
Manzana	Costa Rica	1.831	acre
	Nic. and Salv.	1.727	acre
Marco (= 50 Cast)	Spain and Bol.	.507	lbs.
Maud	India	82.286	lbs.
Mil	Denmark	4.68	mi.
Milla	Nic. and Hond.	1.149	mi.
Morgen	Prussia	.63	acre
Mu	China	.152	acre
Oke	Egypt	2.723	lbs.
	Greece	2.840	lbs.
	Hungary	3.082	lbs.
	Turkey	2.828	lbs.
	Roumania	2.50	pints
Pic	Egypt	21.25	in.
	Turkey	27.90	in.
Picul	Borneo & Celebes	135.64	in.
	China, Japan and Sumatra	133.33	lbs.
	Java	135.10	lbs.
	Philippine Islands	137.90	lbs.
Pie	Argentine Rep.	.948	ft.
	Spain	.914	ft.
Pied	French (usuel)	1.094	ft.
Pood (= 40 funts)	Russia	36.114	lbs.
Pouce (= 12 ligne)	French (usuel)	1.094	in.
Pund	Den. and Swed.	1.102	lbs.
Pu (= 5 Chih)	China	5.249	ft.
Quintal	Argentine Rep.	101.42	lbs.
	Brazil	130.06	lbs.
	Castile, Chili, Mexico and Peru	101.41	lbs.
	Greece	123.20	lbs.
	Newfoundland	112.	lbs.
	Paraguay	100.	lbs.
	Syria	125.	lbs.
Ri	Japan	2.44	mi.

Rottle	Palestine	6.00	lbs.
	Syria	5.75	lbs.
Sagene (= 3 Archines)	Russia	7.00	ft.
Salma	Malta	490.	lbs.
Se	Japan	.0245	acre
Seer	India	29.	oz.
Shaku	Japan	.994	ft.
Sheng	China	1.094	qts.
Sho	Japan	1.60	qts.
Suerte	Uruguay	2700.	cuadras
Sun	Japan	1.193	in.
Tael	Cochin China	590.75	troy gr.
Tan	Japan	.25	acre
		9884.00	sq. ft.
To	Japan	2.	pks.
Toise (= 6 pieds)	French (usuel)	6.562	ft.
Tonda	Denmark	3.948	bu.
Tondeland	Denmark	1.360	acre
Tsubo	Japan	98.84	sq. ft.
Tsun	China	1.26	in.
Tunna	Sweden	4.50	bu.
Tunnland	Sweden	1.22	acre
Vara	Argetine Rep.	34.121	in.
	Cent. Amer.	32.870	in.
	California	33.372	in.
	Chile and Peru	33.367	in.
	Cuba and Venez.	32.384	in.
	Curacao	32.375	in.
	Mexico	33.000	in.
	Paraguay	34.000	in.
	Spain	32.910	in.
Vedro	Russia	3.249	gal.
Vergees	Id. Jersey	71.10	sq. rd.
Verst	Russia	.663	mi.
Vlocka	Russian Poland	41.98	acre

Miscellaneous

Earth's Equatorial Radius	3,962.57 mi.
Earth's Polar Radius	3,949.67 mi.
Equatorial Degree of Latitude	68.70 mi. (statute)
Equatorial Degree of Longitude	69.17 mi. (statute)
Diameter of Sun	864,367.36 mi. (Auwers)
Diameter of Moon	2,159.82 mi. (Newcomb)
Distance to Sun	93. million mi.
Distance to Moon	240,000. mi.
1 Admiralty Knot (1' of earth's circum.)	1.15155 mi. = 1.853 Km
1 Nautical Mile	6,080.27 ft.
1 B. T. U = 1 Pound-degree-Fahr.	778.1 ft. lbs.
1 Calorie = 1 Kilo-degree-Cent.	3,087.35 ft lbs.
1 Kilowatt	44,256.70 ft. lbs.
1 H-P (= 746 Watts)	550. ft. lbs. per sec.
1 Atmosphere, sea level, (76 cm Mercury)	14.7 lbs. per sq. in.
Seconds Pendulum, (N. Y.)	39.1017 in. = .99306 m
Acceleration due to Gravity, (g) at 37°30' Lat.	32.1595 ft. per sec.
$g = 981$ cm per sec$^2 = 977.990 + 5.22 \sin^2$ (lat.)	
$g = 32.2$ ft. per sec.$^2 = 32.087 + 0.171 \sin^2$ (lat.)	
1 Ft. Head of Water	.434 lbs. per sq. in.
1 Lb. Water pressure per sq. in.	2.304 ft. head
1 Cu. Ft. Water	62.5 lbs.
1 Cu. Ft. of Dry Air, 32° F. 30 in.	.0807 lbs.
1 Av. Lb. (=0.4536 liters)	27.68 cu. in water
1 Karat	3.171 — 3.2 grains
1° per mile	92.154 ft.
1' per mile	18.43 in.
1 Mil	.001 in. = .0254 mm
Smallest Resolving Angle, normal vision	1' 12" (Helmholtz)
Side of Equalateral Tri. same area as a circle	1.34677 D
Diameter of a Circle, same area as a square	1.12838 S
Diagonal of a Square	1.41421 S
Coefficient of Expansion for steel tapes	.0000065 per 1° F
Velocity of sound in air	1090. ft. per sec.

SPECIFIC GRAVITY AND WEIGHT OF STONES, BRICK, CEMENT, ETC. (Pure Water=1.00.)

	Sp. Gr.	Lb. per Cu. Ft.
Asphaltum,	1.39	87
Brick, Soft,	1.6	100
Brick, Common,	1.79	112
Brick, Hard,	2.0	125
Brick, Pressed,	2.16	135
Brick, Fire,	2.24 to 2.4	140 to 150
Brick, Sand-lime,	2.18	136
Brickwork in mortar,	1.6	100
Brickwork in cement,	1.79	112
Cement, American, natural,	2.8 to 3.2	...
Cement, Portland,	3.05 to 3.15	...
Cement, Portland, loose,	....	92
Cement, Portland, in barrels,	....	115
Clay,	1.92 to 2.4	120 to 150
Concrete,	1.92 to 2.48	120 to 155
Earth, loose,	1.15 to 1.28	72 to 80
Earth, rammed,	1.44 to 1.76	90 to 110
Emery,	4.	250
Glass,	2.5 to 2.75	156 to 172
Glass, flint,	2.88 to 3.14	180 to 196
Gneiss / Granite	2.56 to 2.72	160 to 170
Gravel,	1.6 to 1.92	100 to 120
Gypsum,	2.08 to 2.4	130 to 150
Hornblende,	3.2 to 3.52	200 to 220
Ice,	0.88 to 0.92	55 to 57
Lime, quick, in bulk,	0.8 to 0.96	50 to 60
Limestone,	2.30 to 2.90	140 to 185
Magnesia, Carbonate,	2.4	150
Marble,	2.56 to 2.88	160 to 180
Masonry, dry rubble,	2.24 to 2.56	140 to 160
Masonry, dressed,	2.24 to 2.88	140 to 180
Mica,	2.80	175
Mortar,	1.44 to 1.6	90 to 100
Mud, soft flowing,	1.67 to 1.92	104 to 120
Pitch,	1.15	72
Plaster of Paris,	1.50 to 1.81	93 to 113
Quartz,	2.64	165
Sand,	1.44 to 1.76	90 to 110
Sand, wet,	1.89 to 2.07	118 to 129
Sandstone,	2.24 to 2.4	140 to 150
Slate,	2.72 to 2.88	170 to 180
Soapstone,	2.65 to 2.8	166 to 175
Stone, various,	2.16 to 3.4	135 to 200
Trap,	2.72 to 3.4	170 to 200
Tile,	1.76 to 1.92	110 to 120

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SPECIFIC GRAVITY AND WEIGHT OF METALS

	Specific Gravity. Range According to Several Authorities	Specific Gravity. Approx. Mean Value, used in Calculation of Weight	Weight per Cubic Foot Lbs.	Weight per Cubic Inch Lbs.
Aluminum,	2.56 to 2.71	2.67	166.5	0.0963
Antimony,	6.66 to 6.86	6.76	421.6	0.2439
Bismuth,	9.74 to 9.90	9.82	612.4	0.3544
Brass: Copper + Zinc				
80 20	7.8 to 8.6	8.60	536.3	0.3103
70 30		8.40	523.8	0.3031
60 40		8.36	521.3	0.3017
50 50		8.20	511.4	0.2959
Bronze { Cop., 95 to 80; Tin, 5 to 20 }	8.52 to 8.96	8.853	552.	0.3195
Cadmium,	8.6 to 8.7	8.65	539.	0.3121
Calcium,	1.58	1.58	98.5	0.0570
Chromium,	5.0	5.0	311.8	0.1804
Cobalt,	8.5 to 8.6	8.55	533.1	0.3085
Gold, pure,	19.245 to 19.361	19.258	1200.9	0.6949
Copper,	8.69 to 8.92	8.853	552.	0.3195
Iridium,	22.38 to 23.	22.38	1396.	0.8076
Iron, Cast,	6.85 to 7.48	7.218	450.	0.2604
Iron, Wrought,	7.4 to 7.9	7.70	480.	0.2779
Lead,	11.07 to 11.44	11.38	709.7	0.4106
Manganese,	7. to 8.	8.	499.	0.2887
Magnesium,	1.69 to 1.75	1.75	109.	0.0641
Mercury, 32°	13.60 to 13.62	13.62	849.3	0.4915
Mercury, 60°	13.58	13.58	846.8	0.4900
Mercury, 212°	13.37 to 13.38	13.38	834.4	0.4828
Nickel,	8.279 to 8.93	8.8	548.7	0.3175
Platinum,	20.33 to 22.07	21.5	1347.0	0.7758
Potassium,	0.865	0.865	53.9	0.0312
Silver,	10.474 to 10.511	10.505	655.1	0.3791
Sodium,	0.97	0.97	60.5	0.0350
Steel,	7.69* to 7.932†	7.854	489.6	0.2834
Tin,	7.291 to 7.409	7.350	458.3	0.2652
Titanium,	5.3	5.3	330.5	0.1913
Tungsten,	17. to 17.6	17.3	1078.7	0.6243
Zinc,	6.86 to 7.20	7.00	436.5	0.2526

* Hard and burned.
† Very pure and soft. The sp. gr. decreases as the carbon is increased.
In the first column of figures the lowest are usually those of cast metals, which are more or less porous; the highest are of metals finely rolled or drawn into wire.

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F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
-40	-40.	26	-3.3	92	33.3	158	70.	224	106.7	290	143.3	360	182.2
-39	-39.4	27	-2.8	93	33.9	159	70.6	225	107.2	291	143.9	370	187.8
-38	-38.9	28	-2.2	94	34.4	160	71.1	226	107.8	292	144.4	380	193.3
-37	-38.3	29	-1.7	95	35.	161	71.7	227	108.3	293	145.	390	198.9
-36	-37.8	30	-1.1	96	35.6	162	72.2	228	108.9	294	145.6	400	204.4
-35	-37.2	31	-0.6	97	36.1	163	72.8	229	109.4	295	146.1	410	210.
-34	-36.7	32	0.	98	36.7	164	73.3	230	110.	296	146.7	420	215.6
-33	-36.1	33	+0.6	99	37.2	165	73.9	231	110.6	297	147.2	430	221.1
-32	-35.6	34	1.1	100	37.8	166	74.4	232	111.1	298	147.8	440	226.7
-31	-35.	35	1.7	101	38.3	167	75.	233	111.7	299	148.3	450	232.2
-30	-34.4	36	2.2	102	38.9	168	75.6	234	112.2	300	148.9	460	237.8
-29	-33.9	37	2.8	103	39.4	169	76.1	235	112.8	301	149.4	470	243.3
-28	-33.3	38	3.3	104	40.	170	76.7	236	113.3	302	150.	480	248.9
-27	-32.8	39	3.9	105	40.6	171	77.2	237	113.9	303	150.6	490	254.4
-26	-32.2	40	4.4	106	41.1	172	77.8	238	114.4	304	151.1	500	260.
-25	-31.7	41	5.	107	41.7	173	78.3	239	115.	305	151.7	510	265.6
-24	-31.1	42	5.6	108	42.2	174	78.9	240	115.6	306	152.2	520	271.1
-23	-30.6	43	6.1	109	42.8	175	79.4	241	116.1	307	152.8	530	276.7
-22	-30.	44	6.7	110	43.3	176	80.	242	116.7	308	153.3	540	282.2
-21	-29.4	45	7.2	111	43.9	177	80.6	243	117.2	309	153.9	550	287.8
-20	-28.9	46	7.8	112	44.4	178	81.1	244	117.8	310	154.4	560	293.3
-19	-28.3	47	8.3	113	45.	179	81.7	245	118.3	311	155.	570	298.9
-18	-27.8	48	8.9	114	45.6	180	82.2	246	118.9	312	155.6	580	304.4
-17	-27.2	49	9.4	115	46.1	181	82.8	247	119.4	313	156.1	590	310.
-16	-26.7	50	10.	116	46.7	182	83.3	248	120.	314	156.7	600	315.6
-15	-26.1	51	10.6	117	47.2	183	83.9	249	120.6	315	157.2	610	321.1
-14	-25.6	52	11.1	118	47.8	184	84.4	250	121.1	316	157.8	620	326.7
-13	-25.	53	11.7	119	48.3	185	85.	251	121.7	317	158.3	630	332.2
-12	-24.4	54	12.2	120	48.9	186	85.6	252	122.2	318	158.9	640	337.8
-11	-23.9	55	12.8	121	49.4	187	86.1	253	122.8	319	159.4	650	343.3
-10	-23.3	56	13.3	122	50.	188	86.7	254	123.3	320	160.	660	348.9
- 9	-22.8	57	13.9	123	50.6	189	87.2	255	123.9	321	160.6	670	354.4
- 8	-22.2	58	14.4	124	51.1	190	87.8	256	124.4	322	161.1	680	360.
- 7	-21.7	59	15.	125	51.7	191	88.3	257	125.	323	161.7	690	365.6
- 6	-21.1	60	15.6	126	52.2	192	88.9	258	125.6	324	162.2	700	371.1
- 5	-20.6	61	16.1	127	52.8	193	89.4	259	126.1	325	162.8	710	376.7
- 4	-20.	62	16.7	128	53.3	194	90.	260	126.7	326	163.3	720	382.2
- 3	-19.4	63	17.2	129	53.9	195	90.6	261	127.2	327	163.9	730	387.8
- 2	-18.9	64	17.8	130	54.4	196	91.1	262	127.8	328	164.4	740	393.3
- 1	-18.3	65	18.3	131	55.	197	91.7	263	128.3	329	165.	750	398.9
0	-17.8	66	18.9	132	55.6	198	92.2	264	128.9	330	165.6	760	404.4
+ 1	-17.2	67	19.4	133	56.1	199	92.8	265	129.4	331	166.1	770	410.
2	-16.7	68	20.	134	56.7	200	93.3	266	130.	332	166.7	780	415.6
3	-16.1	69	20.6	135	57.2	201	93.9	267	130.6	333	167.2	790	421.1
4	-15.6	70	21.1	136	57.8	202	94.4	268	131.1	334	167.8	800	426.7
5	-15.	71	21.7	137	58.3	203	95.	269	131.7	335	168.3	810	432.2
6	-14.4	72	22.2	138	58.9	204	95.6	270	132.2	336	168.9	820	437.8
7	-13.9	73	22.8	139	59.4	205	96.1	271	132.8	337	169.4	830	443.3
8	-13.3	74	23.3	140	60.	206	96.7	272	133.3	338	170.	840	448.9
9	-12.8	75	23.9	141	60.6	207	97.2	273	133.9	339	170.6	850	454.4
10	-12.2	76	24.4	142	61.1	208	97.8	274	134.4	340	171.1	860	460.
11	-11.7	77	25.	143	61.7	209	98.3	275	135.	341	171.7	870	465.6
12	-11.1	78	25.6	144	62.2	210	98.9	276	135.6	342	172.2	880	471.1
13	-10.6	79	26.1	145	62.8	211	99.4	277	136.1	343	172.8	890	476.7
14	-10.	80	26.7	146	63.3	212	100.	278	136.7	344	173.3	900	482.2
15	- 9.4	81	27.2	147	63.9	213	100.6	279	137.2	345	173.9	910	487.8
16	- 8.9	82	27.8	148	64.4	214	101.1	280	137.8	346	174.4	920	493.3
17	- 8.3	83	28.3	149	65.	215	101.7	281	138.3	347	175.	930	498.9
18	- 7.8	84	28.9	150	65.6	216	102.2	282	138.9	348	175.6	940	504.4
19	- 7.2	85	29.4	151	66.1	217	102.8	283	139.4	349	176.1	950	510.
20	- 6.7	86	30.	152	66.7	218	103.3	284	140.	350	176.7	960	515.6
21	- 6.1	87	30.6	153	67.2	219	103.9	285	140.6	351	177.2	970	521.1
22	- 5.6	88	31.1	154	67.8	220	104.4	286	141.1	352	177.8	980	526.7
23	- 5.	89	31.7	155	68.3	221	105.	287	141.7	353	178.3	990	532.2
24	- 4.4	90	32.2	156	68.9	222	105.6	288	142.2	354	178.9	1000	537.8
25	- 3.9	91	32.8	157	69.4	223	106.1	289	142.8	355	179.4	1010	543.3

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C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
-40	-40.	26	78.8	92	197.6	158	316.4	224	435.2	290	554	950	1742
-39	-38.2	27	80.6	93	199.4	159	318.2	225	437.	300	572	960	1760
-38	-36.4	28	82.4	94	201.2	160	320.	226	438.8	310	590	970	1778
-37	-34.6	29	84.2	95	203.	161	321.8	227	440.6	320	608	980	1796
-36	-32.8	30	86.	96	204.8	162	323.6	228	442.4	330	626	990	1814
-35	-31.	31	87.8	97	206.6	163	325.4	229	444.2	340	644	1000	1832
-34	-29.2	32	89.6	98	208.4	164	327.2	230	446.	350	662	1010	1850
-33	-27.4	33	91.4	99	210.2	165	329.	231	447.8	360	680	1020	1868
-32	-25.6	34	93.2	100	212.	166	330.8	232	449.6	370	698	1030	1886
-31	-23.8	35	95.	101	213.8	167	332.6	233	451.4	380	716	1040	1904
-30	-22.	36	96.8	102	215.6	168	334.4	234	453.2	390	734	1050	1922
-29	-20.2	37	98.6	103	217.4	169	336.2	235	455.	400	752	1060	1940
-28	-18.4	38	100.4	104	219.2	170	338.	236	456.8	410	770	1070	1958
-27	-16.6	39	102.2	105	221.	171	339.8	237	458.6	420	788	1080	1976
-26	-14.8	40	104.	106	222.8	172	341.6	238	460.4	430	806	1090	1994
-25	-13.	41	105.8	107	224.6	173	343.4	239	462.2	440	824	1100	2012
-24	-11.2	42	107.6	108	226.4	174	345.2	240	464.	450	842	1110	2030
-23	- 9.4	43	109.4	109	228.2	175	347.	241	465.8	460	860	1120	2048
-22	- 7.6	44	111.2	110	230.	176	348.8	242	467.6	470	878	1130	2066
-21	- 5.8	45	113.	111	231.8	177	350.6	243	469.4	480	896	1140	2084
-20	- 4.	46	114.8	112	233.6	178	352.4	244	471.2	490	914	1150	2102
-19	- 2.2	47	116.6	113	235.4	179	354.2	245	473.	500	932	1160	2120
-18	- 0.4	48	118.4	114	237.2	180	356.	246	474.8	510	950	1170	2138
-17	+ 1.4	49	120.2	115	239.	181	357.8	247	476.6	520	968	1180	2156
-16	3.2	50	122.	116	240.8	182	359.6	248	478.4	530	986	1190	2174
-15	5.	51	123.8	117	242.6	183	361.4	249	480.2	540	1004	1200	2192
-14	6.8	52	125.6	118	244.4	184	363.2	250	482.	550	1022	1210	2210
-13	8.6	53	127.4	119	246.2	185	365.	251	483.8	560	1040	1220	2228
-12	10.4	54	129.2	120	248.	186	366.8	252	485.6	570	1058	1230	2246
-11	12.2	55	131.	121	249.8	187	368.6	253	487.4	580	1076	1240	2264
-10	14.	56	132.8	122	251.6	188	370.4	254	489.2	590	1094	1250	2282
- 9	15.8	57	134.6	123	253.4	189	372.2	255	491.	600	1112	1260	2300
- 8	17.6	58	136.4	124	255.2	190	374.	256	492.8	610	1130	1270	2318
- 7	19.4	59	138.2	125	257.	191	375.8	257	494.6	620	1148	1280	2336
- 6	21.2	60	140.	126	258.8	192	377.6	258	496.4	630	1166	1290	2354
- 5	23.	61	141.8	127	260.6	193	379.4	259	498.2	640	1184	1300	2372
- 4	24.8	62	143.6	128	262.4	194	381.2	260	500.	650	1202	1310	2390
- 3	26.6	63	145.4	129	264.2	195	383.	261	501.8	660	1220	1320	2408
- 2	28.4	64	147.2	130	266.	196	384.8	262	503.6	670	1238	1330	2426
- 1	30.2	65	149.	131	267.8	197	386.6	263	505.4	680	1256	1340	2444
0	32.	66	150.8	132	269.6	198	388.4	264	507.2	690	1274	1350	2462
+ 1	33.8	67	152.6	133	271.4	199	390.2	265	509.	700	1292	1360	2480
2	35.6	68	154.4	134	273.2	200	392.	266	510.8	710	1310	1370	2498
3	37.4	69	156.2	135	275.	201	393.8	267	512.6	720	1328	1380	2516
4	39.2	70	158.	136	276.8	202	395.6	268	514.4	730	1346	1390	2534
5	41.	71	159.8	137	278.6	203	397.4	269	516.2	740	1364	1400	2552
6	42.8	72	161.6	138	280.4	204	399.2	270	518.	750	1382	1410	2570
7	44.6	73	163.4	139	282.2	205	401.	271	519.8	760	1400	1420	2588
8	46.4	74	165.2	140	284.	206	402.8	272	521.6	770	1418	1430	2606
9	48.2	75	167.	141	285.8	207	404.6	273	523.4	780	1436	1440	2624
10	50.	76	168.8	142	287.6	208	406.4	274	525.2	790	1454	1450	2642
11	51.8	77	170.6	143	289.4	209	408.2	275	527.	800	1472	1460	2660
12	53.6	78	172.4	144	291.2	210	410.	276	528.8	810	1490	1470	2678
13	55.4	79	174.2	145	293.	211	411.8	277	530.6	820	1508	1480	2696
14	57.2	80	176.	146	294.8	212	413.6	278	532.4	830	1526	1490	2714
15	59.	81	177.8	147	296.6	213	415.4	279	534.2	840	1544	1500	2732
16	60.8	82	179.6	148	298.4	214	417.2	280	536.	850	1562	1510	2750
17	62.6	83	181.4	149	300.2	215	419.	281	537.8	860	1580	1520	2768
18	64.4	84	183.2	150	302.	216	420.8	282	539.6	870	1598	1530	2786
19	66.2	85	185.	151	303.8	217	422.6	283	541.4	880	1616	1540	2804
20	68.	86	186.8	152	305.6	218	424.4	284	543.2	890	1634	1550	2822
21	69.8	87	188.6	153	307.4	219	426.2	285	545.	900	1652	1600	2912
22	71.6	88	190.4	154	309.2	220	428.	286	546.8	910	1670	1650	3002
23	73.4	89	192.2	155	311.	221	429.8	287	548.6	920	1688	1700	3092
24	75.2	90	194.	156	312.8	222	431.6	288	550.4	930	1706	1750	3182
25	77.	91	195.8	157	314.6	223	433.4	289	552.2	940	1724	1800	3272

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Mean Refraction, (R_M)

Bar., 760 mm = 29.9 in. Temp. 10°C = 50°F.

Approx. Dif. for 1′ can be figured by interpolation.

Alt.	Rm	Alt.	Rm	Alt.	Rm	Alt.	Rm	Alt.	Rm
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 00	34 08.6	7 00	7 24.2	19 00	2 47.6	33 00	1 29.4	52 30	0 44.7
10	32 15.9	10	7 14.9	20	2 44.6	20	1 28.2	53 00	0 43.9
20	30 31.1	20	7 06.0	40	2 41.6	40	1 27.1	30	0 43.1
30	28 53.9	30	6 57.4	20 00	2 38.7	34 00	1 26.1	54 00	0 42.3
40	27 18.2	40	6 49.1	20	2 35.9	20	1 25.0	30	0 41.6
50	25 49.8	50	6 41.2	40	2 33.2	40	1 24.0	55 00	0 40.8
1 00	24 28.3	8 00	6 33.5	21 00	2 30.6	35 00	1 23.0	30	0 40.0
10	23 13.5	10	6 26.0	20	2 28.1	20	1 22.0	56 00	0 39.3
20	22 04.9	20	6 18.9	40	2 25.6	40	1 21.0	57 00	0 37.8
30	21 01.8	30	6 12.0	22 00	2 23.2	36 00	1 20.0	58 00	0 36.4
40	20 03.7	40	6 05.3	20	2 20.9	30	1 18.5	59 00	0 35.0
50	19 09.8	50	5 58.9	40	2 18.6	37 00	1 17.1	60 00	0 33.6
2 00	18 19.7	9 00	5 52.7	23 00	2 16.4	30	1 15.7	61 00	0 32.3
10	17 33.1	20	5 40.8	20	2 14.2	38 00	1 14.4	62 00	0 31.0
20	16 49.7	40	5 29.7	40	2 12.1	30	1 13.1	63 00	0 29.7
30	16 09.5	10 00	5 19.2	24 00	2 10.1	39 00	1 11.8	64 00	0 28.4
40	15 32.1	20	5 09.4	20	2 08.1	30	1 10.5	65 00	0 27.2
50	14 57.1	40	5 00.1	40	2 06.1	40 00	1 09.3	66 00	0 25.9
3 00	14 24.3	11 00	4 51.2	25 00	2 04.2	30	1 08.1	67 00	0 24.7
10	13 53.6	20	4 42.8	20	2 02.4	41 00	1 06.9	68 00	0 23.6
20	13 24.8	40	4 35.0	40	2 00.6	30	1 05.7	69 00	0 22.4
30	12 57.8	12 00	4 27.5	26 00	1 58.8	42 00	1 04.6	70 00	0 21.2
40	12 32.5	20	4 20.3	20	1 57.1	30	1 03.5	71 00	0 20.1
50	12 08.7	40	4 13.5	40	1 55.4	43 00	1 02.4	72 00	0 18.9
4 00	11 46.0	13 00	4 07.1	27 00	1 53.8	30	1 01.3	73 00	0 17.8
10	11 24.6	20	4 00.9	20	1 52.2	44 00	1 00.2	74 00	0 16.7
20	11 04.2	40	3 55.1	40	1 50.6	30	0 59.2	75 00	0 15.6
30	10 44.9	14 00	3 49.5	28 00	1 49.1	45 00	0 58.2	76 00	0 14.5
40	10 26.5	20	3 44.2	20	1 47.6	30	0 57.2	77 00	0 13.5
50	10 09.1	40	3 39.1	40	1 46.1	46 00	0 56.2	78 00	0 12.4
5 00	9 52.6	15 00	3 34.1	29 00	1 44.6	30	0 55.2	79 00	0 11.3
10	9 36.9	20	3 29.4	20	1 43.2	47 00	0 54.2	80 00	0 10.3
20	9 21.9	40	3 24.8	40	1 41.8	30	0 53.3	81 00	0 09.2
30	9 07.6	16 00	3 20.4	30 00	1 40.5	48 00	0 52.5	82 00	0 08.2
40	8 54.0	20	3 16.1	20	1 39.1	30	0 51.6	83 00	0 07.2
50	8 41.0	40	3 12.0	40	1 37.8	49 00	0 50.7	84 00	0 06.1
6 00	8 28.6	17 00	3 08.2	31 00	1 36.6	30	0 49.8	85 00	0 05.1
10	8 16.7	20	3 04.5	20	1 35.3	50 00	0 48.9	86 00	0 04.1
20	8 05.3	40	3 00.9	40	1 34.1	30	0 48.0	87 00	0 03.1
30	7 54.3	18 00	2 57.4	32 00	1 32.0	51 00	0 47.2	88 00	0 02.0
40	7 43.9	20	2 54.0	20	1 31.8	30	0 46.3	89 00	0 01.0
50	7 33.9	40	2 50.7	40	1 30.6	52 00	0 45.5	90 00	0 00.0

Table of Aneroid Altitudes, 50° F.

(After Airey)

$$D_A = 62759 (\log h - \log H) \left(1 + \frac{t' + t - 100}{1000}\right)$$

The first factor in the second half contains the corrected height of a mercury column at both lower and upper stations. The second factor is the temperature correction for the following table.

RULE: Take difference in altitude readings and correct for temperature by increasing it by $\frac{1}{1000}$ of itself for each degree that the sum of the temperatures at both stations exceeds 100°F.

Cor. Bar.	Height	Cor. Bar.	Height	Cor. Bar.	Height	Cor. Bar.	Height	Cor. Bar.	Height
in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.
31.00	0	28.28	2500	25.80	5000	23.54	7500	21.47	10000
30.94	50	28.23	2550	25.75	5050	23.50	7550	21.44	10050
30.88	100	28.18	2600	25.71	5100	23.45	7600	21.40	10100
30.83	150	28.12	2650	25.66	5150	23.41	7650	21.36	10150
30.77	200	28.07	2700	25.61	5200	23.37	7700	21.32	10200
30.71	250	28.02	2750	25.56	5250	23.32	7750	21.28	10250
30.66	300	27.97	2800	25.52	5300	23.28	7800	21.24	10300
30.60	350	27.92	2850	25.47	5350	23.24	7850	21.20	10350
30.54	400	27.87	2900	25.42	5400	23.20	7900	21.16	10400
30.49	450	27.82	2950	25.38	5450	23.15	7950	21.12	10450
30.43	500	27.76	3000	25.33	5500	23.11	8000	21.08	10500
30.38	550	27.71	3050	25.28	5550	23.07	8050	21.05	10550
30.32	600	27.66	3100	25.24	5600	23.03	8100	21.01	10600
30.26	650	27.61	3150	25.19	5650	22.98	8150	20.97	10650
30.21	700	27.56	3200	25.15	5700	22.94	8200	20.93	10700
30.15	750	27.51	3250	25.10	5750	22.90	8250	20.89	10750
30.10	800	27.46	3300	25.05	5800	22.86	8300	20.85	10800
30.04	850	27.41	3350	25.01	5850	22.82	8350	20.82	10850
29.99	900	27.36	3400	24.96	5900	22.77	8400	20.78	10900
29.93	950	27.31	3450	24.92	5950	22.73	8450	20.74	10950
29.88	1000	27.26	3500	24.87	6000	22.69	8500	20.70	11000
29.82	1050	27.21	3550	24.82	6050	22.65	8550	20.66	11050
29.77	1100	27.16	3600	24.78	6100	22.61	8600	20.63	11100
29.71	1150	27.11	3650	24.73	6150	22.57	8650	20.59	11150
29.66	1200	27.06	3700	24.69	6200	22.52	8700	20.55	11200
29.61	1250	27.01	3750	24.64	6250	22.48	8750	20.51	11250
29.55	1300	26.96	3800	24.60	6300	22.44	8800	20.47	11300
29.50	1350	26.91	3850	24.55	6350	22.40	8850	20.44	11350
29.44	1400	26.86	3900	24.51	6400	22.36	8900	20.40	11400
29.39	1450	26.81	3950	24.46	6450	22.32	8950	20.36	11450
29.34	1500	26.76	4000	24.42	6500	22.28	9000	20.32	11500
29.28	1550	26.72	4050	24.37	6550	22.24	9050	20.29	11550
29.23	1600	26.67	4100	24.33	6600	22.20	9100	20.25	11600
29.17	1650	26.62	4150	24.28	6650	22.16	9150	20.21	11650
29.12	1700	26.57	4200	24.24	6700	22.11	9200	20.18	11700
29.07	1750	26.52	4250	24.20	6750	22.07	9250	20.14	11750
29.01	1800	26.47	4300	24.15	6800	22.03	9300	20.10	11800
28.96	1850	26.42	4350	24.11	6850	21.99	9350	20.07	11850
28.91	1900	26.37	4400	24.06	6900	21.95	9400	20.03	11900
28.86	1950	26.33	4450	24.02	6950	21.91	9450	19.99	11950
28.80	2000	26.28	4500	23.97	7000	21.87	9500	19.95	12000
28.75	2050	26.23	4550	23.93	7050	21.83	9550	19.241	13000
28.70	2100	26.18	4600	23.89	7100	21.79	9600	18.548	14000
28.64	2150	26.13	4650	23.84	7150	21.75	9650	17.880	15000
28.59	2200	26.09	4700	23.80	7200	21.71	9700	17.235	16000
28.54	2250	26.04	4750	23.76	7250	21.67	9750	16.615	17000
28.49	2300	25.99	4800	23.71	7300	21.63	9800	16.016	18000
28.43	2350	25.94	4850	23.67	7350	21.59	9850	15.439	19000
28.38	2400	25.89	4900	23.62	7400	21.55	9900	14.883	20000
28.33	2450	25.85	4950	23.58	7450	21.51	9950		

Conversion Table

′	″	Ft. per mile	Per 100	′	″	Ft. per mile	Per 100	′	″	Ft. per mile	Per 100
0	21	.53	.01	17	32	26.93	.51	34	43	53.33	1.01
0	41	1.06	.02	17	53	27.46	.52	35	04	53.86	1.02
1	02	1.58	.03	18	13	27.98	.53	35	24	54.38	1.03
1	23	2.11	.04	18	34	28.51	.54	35	45	54.91	1.04
1	43	2.64	.05	18	54	29.04	.55	36	05	55.44	1.05
2	04	3.17	.06	19	15	29.57	.56	36	26	55.97	1.06
2	24	3.70	07	19	36	30.10	.57	36	47	56.50	1.07
2	45	4.22	.08	19	56	30.62	.58	37	08	57.02	1.08
3	06	4.75	.09	20	17	31.15	.59	37	28	57.55	1.09
3	26	5.28	.10	20	38	31.68	.60	37	49	58.08	1.10
3	47	5.81	.11	20	58	32.21	.61	38	09	58.61	1.11
4	08	6.34	.12	21	19	32.74	.62	38	30	59.14	1.12
4	28	6.86	.13	21	39	33.26	.63	38	51	59.66	1.13
4	49	7.39	.14	22	00	34.79	.64	39	11	60.19	1.14
5	09	7.92	.15	22	21	34.32	.65	39	32	60.72	1.15
5	30	8.45	.16	22	41	34.85	.66	39	53	61.25	1.16
5	51	8.98	.17	23	02	35.38	.67	40	13	61.78	1.17
6	11	9.50	.18	23	23	35.90	.68	40	34	62.30	1.18
6	32	10.03	.19	23	43	36.43	.69	40	54	62.83	1.19
6	53	10.56	.20	24	04	36.96	.70	41	15	63.36	1.20
7	13	11.09	.21	24	24	37.49	.71	41	35	63.89	1.21
7	34	11.62	.22	24	45	38.02	.72	41	56	64.42	1.22
7	54	12.14	.23	25	06	38.54	.73	42	17	64.94	1.23
8	15	12.67	.24	25	26	39.07	.74	42	38	65.47	1.24
8	36	13.20	.25	25	47	39.60	.75	42	58	66.00	1.25
8	56	13.73	.26	26	08	40.13	.76	43	19	66.53	1.26
9	17	14.26	.27	26	28	40.66	.77	43	39	67.06	1.27
9	38	14.78	.28	26	49	41.18	.78	44	00	67.58	1.28
9	58	15.31	.29	27	09	41.71	.79	44	21	68.11	1.29
10	19	15.84	.30	27	30	42.24	.80	44	41	68.64	1.30
10	39	16.37	.31	27	51	42.77	.81	45	02	69.17	1.31
11	00	16.90	.32	28	11	43.30	.82	45	23	69.70	1.32
11	21	17.42	.33	28	32	43.82	.83	45	43	70.22	1.33
11	41	17.95	.34	28	53	44.35	.84	46	04	70.75	1.34
12	02	18.48	.35	29	13	44.88	.85	46	24	71.28	1.35
12	23	19.01	.36	29	34	45.41	.86	46	45	71.81	1.36
12	43	19.54	.37	29	54	45.94	.87	47	06	72.34	1.37
13	04	20.06	.38	30	15	46.46	.88	47	26	72.86	1.38
13	24	20.59	.39	30	36	46.99	.89	47	47	73.39	1.39
13	45	21.12	.40	30	57	47.52	.90	48	08	73.92	1.40
14	06	21.65	.41	31	17	48.05	.91	48	28	74.45	1.41
14	26	22.18	.42	31	38	48.58	.92	48	49	74.98	1.42
14	47	22.70	.43	30	58	49.10	.93	49	09	75 50	1.43
15	08	23.23	.44	32	19	49.63	.94	49	30	76.03	1.44
15	28	23.76	.45	32	39	50.16	.95	49	51	76.56	1.45
15	49	24.29	.45	33	00	50.69	.96	50	11	77.09	1.46
16	09	24.82	.47	33	21	51.22	.97	50	32	77.62	1.47
16	30	25.34	.48	33	41	51.74	.98	50	52	78.14	1.48
16	51	25.87	.49	34	02	52.27	.99	51	13	78.67	1.49
17	11	26.40	.50	34	23	52.80	1.00	51	34	79.20	1.50

Conversion Table

° ′ ″	Ft. per mile	Per 100	° ′ ″	Ft. per mile	Per 100	° ′ ″	Ft. per mile	Per 100
51 54	79.73	1.51	1 10 28	108.24	2.05	2 55 10	209.28	5.10
52 15	80.26	1.52	1 12 11	110.88	2.10	2 58 36	274.56	5.20
52 36	80.78	1.53	1 13 54	113.52	2.15	3 02 09	279.84	5.30
52 56	81.31	1.54	1 15 37	116.16	2.20	3 05 27	285.12	5.40
53 17	81.84	1.55	1 17 20	118.80	2.25	3 08 53	290.40	5.50
53 37	82.37	1.56	1 19 03	121.44	2.30	3 12 19	295.68	5.60
53 58	82.90	1.57	1 20 46	124.08	2.35	3 15 44	300.96	5.70
54 19	83.42	1.58	1 22 29	126.72	2.40	3 19 10	306.24	5.80
54 39	83.95	1.59	1 24 12	129.36	2.45	3 22 36	311.52	5.90
55 00	84.48	1.60	1 25 56	132.00	2.50	**3 26 01**	**316.80**	**6.00**
55 21	85.01	1.61	1 27 39	134.64	2.55	3 29 27	322.08	6,10
55 41	85.54	1.62	1 29 22	137.28	2.60	3 32 52	327.36	6.20
56 02	86.06	1.63	1 31 05	139.92	2.65	3 36 18	332.64	6.30
56 22	86.59	1.64	1 32 48	142.56	2.70	3 39 43	337.92	6.40
56 43	87.12	1.65	1 34 31	145.20	2.75	3 43 08	343.20	6.50
57 04	87.65	1.66	1 36 14	147.84	2.80	3 46 34	348.48	6.60
57 24	88.18	1.67	1 37 57	150.48	2.85	3 49 59	353.76	6.70
57 45	88.70	1.68	1 39 49	153.12	2.90	3 53 24	359.04	6.80
58 06	89.23	1.69	1 41 23	155.76	2.95	3 56 50	364.32	6.90
58 26	89.76	1.70	**1 43 06**	**158.40**	**3.00**	**4 00 15**	**369.60**	**7.00**
58 47	90.29	1.71	1 44 49	161.04	3.05	4 03 40	374.88	7.10
59 07	90.82	1.72	1 46 32	163.68	3.10	4 07 06	380.16	7.20
59 28	91 34	1.73	1 48 15	166.32	3.15	4 10 31	385.44	7.30
59 49	91.87	1.74	1 49 58	168.96	3.20	4 13 56	390.72	7.40
1 00 09	92 40	1.75	1 51 41	171.60	3.25	4 17 21	396.00	7.50
1 00 30	92.93	1.76	1 53 24	174.24	3.30	4 20 46	401.28	7.60
1 00 51	95.46	1.77	1 55 07	176.88	3.35	4 24 11	406.56	7.70
1 01 11	93.98	1.78	1 56 50	179.52	3.40	4 27 36	411.84	7.80
1 01 32	94.51	1.79	1 58 33	182.16	3.45	4 31 01	417.12	7.90
1 01 52	95.04	1.80	2 09 16	184.80	3.50	**4 34 26**	**422.40**	**8.00**
1 02 13	95.57	1.81	2 01 59	187.44	3.55	4 37 51	427.68	8.10
1 02 34	96.10	1.82	2 03 42	190.08	3.60	4 41 16	432.96	8.20
1 02 54	96.62	1.83	2 05 25	192.72	4.65	4 44 41	438.24	8.30
1 03 15	97.15	1.84	2 07 08	195.36	3.70	4 48 06	443.52	8.40
1 03 35	97.68	1.85	2 08 51	198.00	3.75	4 51 30	448.80	8.50
1 03 56	98.21	1.86	2 10 34	200.64	3.80	4 54 55	454.08	8.60
1 04 17	98.74	1.87	2 12 17	203.28	3.85	4 58 20	459.36	8.70
1 04 37	99.26	1.88	2 14 00	205.92	3.90	5 01 44	464.64	8.80
1 04 58	99.79	1.89	2 15 43	208.56	3.95	5 05 10	469.92	8.90
1 05 19	100.32	1.90	**2 17 26**	**211.20**	**4.00**	**5 08 34**	**475.20**	**9.00**
1 05 39	100.85	1.91	2 20 52	216.48	4.10	5 11 59	480.48	9.10
1 06 00	101.38	1.92	2 24 18	221.76	4.20	5 15 23	485.76	9.20
1 06 20	101.90	1.93	2 27 44	227.04	4.30	5 18 48	491.04	9.30
1 06 41	102.43	1.94	2 31 10	232.32	4.40	5 22 12	496.32	9.40
1 07 02	102.96	1.95	2 34 36	237.60	4.50	5 25 37	501.60	9.50
1 07 22	103.49	1.96	2 38 01	242.88	4.60	5 26 01	506.88	9.60
1 07 43	104.02	1.97	2 41 27	248.16	4.70	5 32 25	512.16	9.70
1 08 04	104.54	1.98	2 44 53	253.44	4.80	5 35 50	517.44	9.80
1 08 24	105.07	1.99	2 48 19	258.72	4.90	5 39 14	522.72	9.90
1 08 45	**105.60**	**2.00**	**2 51 45**	**264.00**	**5.00**	**5 42 38**	**528.00**	**10.00**

Gradient in degrees	Difference of elevation for sloping distances of—								
	1.	2.	3.	4.	5.	6.	7.	8.	9.
½	00087	00174	00262	00349	00436	00523	00611	00698	00785
1	00174	00349	00523	00698	00873	01047	01222	01396	01571
1½	00262	00523	00785	01047	01309	01571	01832	02094	02356
2	00349	00698	01047	01396	01745	02094	02443	02792	03141
2½	00436	00872	01308	01745	02181	02617	03053	03489	03926
3	00523	01047	01570	02093	02617	03140	03663	04187	04710
4	00697	01395	02093	02790	03488	04185	04883	05580	06278
5	00871	01743	02615	03486	04358	05229	06101	06972	07844
6	01045	02090	03136	04181	05226	06272	07317	08362	09407
7	01219	02437	03656	04875	06093	07312	08531	09749	10968
8	01392	02783	04175	05567	06959	08350	09742	11134	12525
9	01564	03129	04693	06257	07822	09386	10950	12515	14079
10	01736	03473	05209	06946	08682	10419	12155	13892	15628
12	02079	04158	06237	08316	10395	12475	14554	16633	18712
14	02419	04838	07258	09677	12096	14515	16934	19354	21773
16	02756	05513	08269	11025	13782	16538	19294	22051	24807
18	03090	06180	09270	12361	15451	18541	21631	24721	27811
20	03420	06840	10261	13681	17101	20521	23941	27362	30781

Rule.—From the line of the given gradient, take out the tabular numbers corresponding to each of the figures of the given distance, *beginning* at the *right*, and set them down; each one place to the left of the one above it. Retain the ciphers at the beginning of the last tabular number taken out, if any. Other left-hand ciphers may be dropped.

Add the tabular numbers and point off from the *left* the number of places equal to that of the left-hand figure of the distance, *counting any left-hand ciphers*. The result is the difference of elevation, in the same unit as the distance.

Example:—For the diff. of elevation corresponding to a gradient of 3° and a distance of 6,273 ft., on the slope—

For 3, opp. 3° and under 3,	1570	
For 7, opp. 3° and under 7,	3663	
For 2, opp. 3° and under 2,	1047	
For 6, opp. 3° and under 6,	03140	(retain leading cipher)
As 6 is in 4th place, point off 4	0328. 2900	
Diff. of elevation =	328. 29 ft.	

Gradient in degrees	Horizontal distances for sloping distance of—								
	1.	2.	3.	4.	5.	6.	7.	8.	9.
1	09998	19997	29995	39994	49992	59991	69989	79988	89986
2	09994	19988	29982	39976	49969	59963	69957	79951	89945
3	09986	19972	29959	39945	49931	59918	69904	79890	89877
4	09976	19951	29927	39902	49878	59854	69829	79805	89781
5	09962	19924	29886	39848	49810	59772	69733	79695	89657
6	09945	19890	29836	39781	49726	59671	69616	79562	89507
7	09925	19851	29776	39702	49627	59553	69478	79404	89329
8	09903	19805	29708	39611	49513	59416	69319	79221	89124
9	09877	19754	29631	39507	49384	59261	69138	79015	88892
10	09848	19696	29544	39392	49240	59088	68936	78785	88633
12	09781	19563	29344	39126	48907	58689	68470	78252	88033
14	09703	19406	29108	38812	48515	58218	67921	77624	87326
16	09613	19225	28838	38450	48063	57676	67288	76901	86513
18	09510	19021	28532	38042	47553	57063	66574	76084	85595
20	09397	18794	28191	37588	46985	56381	65778	75175	84572

Stadia Coefficients, Vertical Rod.

H = $\mathrm{Cos}^2 v$ and $\frac{1}{2}\sin 2v$. = V

Min.	0°		1°		2°		3°		4°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	1.0000	.0000	.9997	.0174	.9988	.0349	.9973	.0523	.9951	.0696
2	1.0000	.0006	.9997	.0180	.9987	.0355	.9972	.0528	.9951	.0702
4	1.0000	.0012	.9997	.0186	.9987	.0360	.9971	.0534	.9950	.0707
6	1.0000	.0017	.9996	.0192	.9987	.0366	.9971	.0540	.9949	.0713
8	1.0000	.0023	.9996	.0198	.9986	.0372	.9970	.0546	.9948	.0719
10	1.0000	.0029	.9996	.0204	.9986	.0378	.9969	.0552	.9947	.0725
12	1.0000	.0035	.9996	.0209	.9985	.0384	.9969	.0557	.9946	.0730
14	1.0000	.0041	.9995	.0215	.9985	.0390	.9968	.0563	.9946	.0736
16	1.0000	.0047	.9995	.0221	.9984	.0395	.9968	.0569	.9945	.0742
18	1.0000	.0052	.9995	.0227	.9984	.0401	.9967	.0575	.9944	.0748
20	1.0000	.0058	.9995	.0233	.9983	.0407	.9966	.0580	.9943	.0753
22	1.0000	.0064	.9994	.0238	.9983	.0413	.9966	.0586	.9942	.0759
24	1.0000	.0070	.9994	.0244	.9982	.0418	.9965	.0592	.9941	.0765
26	.9999	.0076	.9994	.0250	.9982	.0424	.9964	.0598	.9940	.0771
28	.9999	.0081	.9993	.0256	.9981	.0430	.9963	.0604	.9939	.0776
30	.9999	.0087	.9993	.0262	.9981	.0436	.9963	.0609	.9938	.0782
32	.9999	.0093	.9993	.0267	.9980	.0442	.9962	.0615	.9938	.0783
34	.9999	.0099	.9993	.0273	.9980	.0448	.9962	.0621	.9937	.0794
36	.9999	.0105	.9992	.0279	.9979	.0453	.9961	.0627	.9936	.0799
38	.9999	.0111	.9902	.0285	.9979	.0459	.9960	.0633	.9935	.0805
40	.9999	.0116	.9992	.0291	.9978	.0465	.9959	.0638	.9934	.0811
42	.9999	.0122	.9991	.0297	.9978	.0471	.9959	.0644	.9933	.0817
44	.9998	.0128	.9991	.0302	.9977	.0276	.9958	.0650	.9932	.0822
46	.9998	.0134	.9990	.0308	.9977	.0482	.9957	.0656	.9931	.0828
48	.9998	.0140	.9990	.0314	.9976	.0488	.9956	.0661	.9930	.0834
50	.9998	.0145	.9990	.0320	.9976	.0494	.9956	.0667	.9929	.0840
52	.9998	.0151	.9989	.0326	.9975	.0499	.9955	.0673	.9928	.0845
54	.9998	.0157	.9989	.0331	.9974	.0505	.9954	.0678	.9927	.0851
56	.9997	.0163	.9989	.0337	.9974	.0511	.9953	.0684	.9926	.0857
58	.9997	.0169	.9988	.0343	.9973	.0517	.9952	.0690	.9925	.0863
60	.9997	.0174	.9988	.0349	.9973	.0523	.9951	.0696	.9924	.0868
$c+f$										
0.40	0.40	0.00	0.40	0.00	0.40	0.01	0.40	0.02	0.40	0.03
0.60	0.60	0.00	0.60	0.01	0.60	0.02	0.60	0.04	0.60	0.05
0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05	0.75	0.06
1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06	1.00	0.08
1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08	1.25	0.10

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

H = $\text{Cos}^2 v$ and $\frac{1}{2}\sin 2v$ = V

Min.	5° Hor. dist.	5° Diff. elev.	6° Hor. dist.	6° Diff. elev.	7° Hor. dist.	7° Diff. elev.	8° Hor. dist.	8° Diff. elev.	9° Hor. dist.	9° Diff. elev.
0	.9924	.0868	.9891	.1040	.9851	.1210	.9806	.1378	.9755	.1545
2	.9923	.0874	.9890	.1045	.9850	.1215	.9805	.1384	.9753	.1551
4	.9922	.0880	.9888	.1051	.9848	.1221	.9803	.1389	.9752	.1556
6	.9921	.0885	.9887	.1057	.9847	.1226	.9801	.1395	.9750	.1562
8	.9920	.0891	.9886	.1062	.9846	.1232	.9800	.1401	.9748	.1567
10	.9919	.0897	.9885	.1068	.9844	.1238	.9798	.1406	.9746	.1573
12	.9918	.0903	.9883	.1074	.9843	.1243	.9797	.1412	.9744	.1578
14	.9917	.0908	.9882	.1079	.9841	.1249	.9795	.1417	.9743	.1584
16	.9916	.0914	.9881	.1085	.9840	.1255	.9793	.1423	.9741	.1589
18	.9915	.0920	.9880	.1091	.9839	.1260	.9792	.1428	.9739	.1595
20	.9914	.0925	.9878	.1096	.9837	.1266	.9790	.1434	.9737	.1600
22	.9913	.0931	.9877	.1102	.9836	.1272	.9788	.1440	.9735	.1606
24	.9911	.0937	.9876	.1108	.9834	.1277	.9787	.1445	.9733	.1611
26	.9910	.0943	.9874	.1113	.9833	.1283	.9785	.1451	.9731	.1617
28	.9909	.0948	.9873	1119	.9831	.1288	.9783	.1456	.9729	.1622
30	.9908	.0954	.9872	.1125	.9829	.1294	.9782	.1462	.9728	.1628
32	.9907	.0960	.9871	.1130	.9828	.1300	.9780	.1467	.9726	.1633
34	.9906	.0965	.9869	.1136	.9827	.1305	.9778	.1473	.9724	.1639
36	.9905	.0971	.9868	.1142	.9825	.1311	.9776	.1479	.9722	.1644
38	.9904	.0977	.9867	.1147	.9824	.1317	.9775	.1484	.9720	.1650
40	.9903	.0983	.9865	.1153	.9822	.1322	.9773	.1490	.9718	.1655
42	.9901	.0988	.9864	.1159	.9820	.1328	.9771	.1495	.9716	.1661
44	.9900	.0994	.9863	.1164	.9819	.1333	.9769	.1501	.9714	.1666
46	.9899	.1000	.9861	.1170	.9817	.1339	.9768	.1506	.9712	.1672
48	.9898	.1005	.9860	.1176	.9816	.1345	.9766	.1512	.9710	.1677
50	.9897	.1011	.9858	.1181	.9814	.1350	.9764	.1517	.9708	.1683
52	.9896	.1017	.9857	.1187	.9813	.1356	.9762	.1523	.9706	.1688
54	.9894	.1022	.9856	.1193	.9811	.1361	.9761	.1528	.9704	.1694
56	.9893	.1028	.9854	.1198	.9810	.1367	.9759	.1534	.9702	.1699
58	.9892	.1034	.9853	.1204	.9808	.1373	.9757	.15.0	.9700	.1705
60	.9891	.1040	.9851	.1210	.9806	.1378	.9755	.1545	.9698	.1710
$c+f$										
0.40	0.40	0.04	0.40	0.04	0.40	0.04	0.40	0.06	0.40	0.06
0.60	0.60	0.05	0.60	0.06	0.60	0.08	0.60	0.09	0.60	0.10
0.75	0.75	0.07	0.75	0.08	0.74	0.10	0.74	0.11	0.74	0.12
1.00	0.99	0.09	0.99	0.11	0.99	0.13	0.99	0.15	0.99	0.16
1.25	1.24	0.11	1.24	0.14	1.24	0.16	1.23	0.18	1.23	0.21

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

$H = \cos^2 v$ and $\frac{1}{2} \sin 2v = V$

Min.	10°		11°		12°		13°		14°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.9698	.1710	.9636	.1873	.9568	.2034	.9494	.2192	.9415	.2347
2	.9696	.1716	.9634	.1878	.9565	.2039	.9491	.2197	.9412	.2352
4	.9694	.1721	.9632	.1884	.9563	.2044	.9489	.2202	.9409	.2358
6	.9692	.1726	.9629	.1889	.9561	.2050	.9486	.2208	.9407	.2363
8	.9690	.1732	.9627	.1895	.9558	.2055	.9484	.2213	.9404	.2368
10	.9688	.1737	.9625	.1900	.9556	.2060	.9481	.2218	.9401	.2373
12	.9686	.1743	.9623	.1905	.9553	.2066	.9479	.2223	.9398	.2378
14	.9684	.1748	.9621	.1911	.9551	.2071	.9476	.2228	.9395	.2383
16	.9682	.1754	.9618	.1916	.9549	.2076	.9473	.2234	.9393	.2388
18	.9680	.1759	.9616	.1921	.9546	.2081	.9471	.2239	.9390	.2393
20	.9678	.1765	.9614	.1927	.9544	.2087	.9468	.2244	.9387	.2399
22	.9676	.1770	.9612	.1932	.9541	.2092	.9466	.2249	.9384	.2404
24	.9674	.1776	.9609	.1938	.9539	.2097	.9463	.2254	.9381	.2409
26	.9672	.1781	.9607	.1943	.9536	.2103	.9460	.2260	.9379	.2414
28	.9670	.1786	.9605	.1948	.9534	.2108	.9458	.2265	.9376	.2419
30	.9668	.1792	.9603	.1954	.9532	.2113	.9455	.2270	.9373	.2424
32	.9666	.1797	.9600	.1959	.9529	.2118	.9452	.2275	.9370	.2429
34	.9664	.1803	.9598	.1964	.9527	.2124	.9450	.2280	.9367	.2434
36	.9662	.1808	.9596	.1970	.9524	.2129	.9447	.2285	.9365	.2439
38	.9660	.1814	.9593	.1975	.9522	.2134	.9444	.2291	.9362	.2444
40	.9657	.1819	.9591	.1980	.9519	.2139	.9442	.2296	.9359	.2449
42	.9655	.1824	.9589	.1986	.9517	.2145	.9439	.2301	.9356	.2455
44	.9653	.1830	.9586	.1991	.9514	.2150	.9436	.2306	.9353	.2460
46	.9651	.1835	.9584	.1996	.9512	.2155	.9434	.2311	.9350	.2465
48	.9649	.1841	.9582	.2002	.9509	.2160	.9431	.2316	.9347	.2470
50	.9647	.1846	.9579	.2007	.9507	.2166	.9428	.2322	.9345	.2475
52	.9645	.1851	.9577	.2012	.9504	.2171	.9426	.2327	.9342	.2480
54	.9642	.1857	.9575	.2018	.9502	.2176	.9423	.2332	.9339	.2485
56	.9640	.1862	.9572	.2023	.9499	.2181	.9420	.2337	.9336	.2490
58	.9638	.1868	.9570	.2028	.9497	.2187	.9417	.2342	.9333	.2495
60	.9636	.1873	.9568	.2034	.9494	.2192	.9415	.2347	.9330	.2500
$c+f$										
0.40	0.39	0.07	0.39	0.08	0.39	0.08	0.39	0.09	0.39	0.10
0.60	0.59	0.11	0.59	0.12	0.59	0.13	0.58	0.14	0.58	0.15
0.75	0.74	0.14	0.73	0.15	0.73	0.16	0.73	0.17	0.73	0.19
1.00	0.98	0.18	0.98	0.20	0.98	0.22	0.97	0.23	0.97	0.25
1.25	1.23	0.23	1.22	0.25	1.22	0.27	1.21	0.29	1.21	0.31

$(c+f) \cos v$ and $(c+f) \sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

$H = \cos^2 v$ and $\frac{1}{2}\sin 2v = V$

Min.	15°		16°		17°		18°		19°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.9330	.2500	.9240	.2650	.9145	.2796	.9045	.2939	.8940	.3078
2	.9327	.2505	.9237	.2655	.9142	.2801	.9042	.2944	.8936	.3083
4	.9324	.2510	.9234	.2659	.9139	.2806	.9038	.2948	.8933	.3087
6	.9321	.2515	.9231	.2664	.9135	.2810	.9035	.2953	.8929	.3092
8	.9318	.2520	.9228	.2669	.9132	.2815	.9031	.2958	.8926	.3097
10	.9316	.2525	.9225	.2674	.9129	.2820	.9028	.2962	.8922	.3101
12	.9313	.2530	.9222	.2679	.9126	.2825	.9024	.2967	.8918	.3106
14	.9310	.2535	.9219	.2684	.9122	.2830	.9021	.2972	.8915	.3110
16	.9307	.2540	.9215	.2689	.9119	.2834	.9018	.2976	.8911	.3115
18	.9304	.2545	.9212	.2694	.9116	.2839	.9014	.2981	.8908	.3119
20	.9301	.2550	.9209	.2699	.9112	.2844	.9011	.2986	.8904	.3124
22	.9298	.2555	.9206	.2704	.9109	.2849	.9007	.2990	.8900	.3128
24	.9295	.2560	.9203	.2709	.9106	.2854	.9004	.2995	.8896	.3133
26	.9292	.2565	.9200	.2713	.9102	.2858	.9000	.3000	.8893	.3138
28	.9289	.2570	.9197	.2718	.9099	.2863	.8997	.3004	.8889	.3142
30	.9286	.2575	.9193	.2723	.9096	.2868	.8993	.3009	.8886	.3147
32	.9283	.2580	.9190	.2728	.9092	.2873	.8990	.3014	.8882	.3151
34	.9280	.2585	.9187	.2733	.9089	.2877	.8986	.3019	.8878	.3156
36	.9277	.2590	.9184	.2738	.9086	.2882	.8983	.3023	.8875	.3160
38	.9274	.2595	.9181	.2743	.9082	.2887	.8979	.3028	.8871	.3165
40	.9271	.2600	.9177	.2748	.9079	.2892	.8976	.3032	.8867	.3169
42	.9268	.2605	.9174	.2752	.9076	.2896	.8972	.3037	.8864	.3174
44	.9265	.2610	.9171	.2757	.9072	.2901	.8969	.3041	.8860	.3178
46	.9262	.2615	.9168	.2762	.9069	.2906	.8965	.3046	.8856	.3183
48	.9259	.2620	.9165	.2767	.9066	.2911	.8961	.3051	.8853	.3187
50	.9256	.2625	.9161	.2772	.9062	.2915	.8958	.3055	.8849	.3192
52	.9253	.2630	.9158	.2777	.9059	.2920	.8954	.3060	.8845	.3196
54	.9249	.2635	.9155	.2781	.9055	.2925	.8951	.3065	.8841	.3201
56	.9246	.2640	.9152	.2786	.9052	.2930	.8947	.3069	.8838	.3205
58	.9243	.2645	.9148	.2791	.9048	.2934	.8944	.3074	.8834	.3209
60	.9240	.2650	.9145	.2796	.9045	.2939	.8940	.3078	.8830	.3214
$c+f$										
0.40	0.38	0.11	0.38	0.11	0.38	0.12	0.38	0.13	0.38	0.13
0.60	0.58	0.16	0.58	0.17	0.57	0.18	0.57	0.19	0.57	0.20
0.75	0.72	0.20	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
1.00	0.96	0.27	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
1.25	1.20	0.34	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

H = $\cos^2 v$ and $\frac{1}{2}\sin 2v$. = V

Min.	20°		21°		22°		23°		24°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.8830	.3214	.8716	.3346	.8597	.3473	.8473	.3597	.8346	.3716
2	.8826	.3218	.8712	.3350	.8593	.3477	.8469	.3601	.8341	.3720
4	.8823	.3223	.8708	.3354	.8589	.3482	.8465	.3605	.8337	.3723
6	.8819	.3227	.8704	.3359	.8585	.3486	.8461	.3609	.8333	.3727
8	.8815	.3232	.8700	.3363	.8580	.3490	.8457	.3613	.8328	.3731
10	.8811	.3236	.8696	.3367	.8576	.3494	.8452	.3617	.8324	.3735
12	.8808	.3241	.8692	.3372	.8572	.3498	.8448	.3621	.8320	.3739
14	.8804	.3245	.8688	.3376	.8568	.3502	.8444	.3625	.8315	.3743
16	.8800	.3249	.8684	.3380	.8564	.3507	.8440	.3629	.8311	.3747
18	.8796	.3254	.8680	.3384	.8560	.3511	.8435	.3633	.8307	.3751
20	.8793	.3258	.8677	.3389	.8556	.3515	.8431	.3637	.8302	.3754
22	.8789	.3263	.8673	.3393	.8552	.3519	.8427	.3641	.8298	.3758
24	.8785	.3267	.8669	.3397	.8548	.3523	.8423	.3645	.8293	.3762
26	.8781	.3272	.8665	.3401	.8544	.3527	.8418	.3649	.8289	.3766
28	.8777	.3276	.8661	.3406	.8540	.3531	.8414	.3653	.8285	.3770
30	.8774	.3280	.8657	.3410	.8536	.3536	.8410	.3657	.8280	.3774
32	.8770	.3285	.8653	.3414	.8531	.3540	.8406	.3661	.8276	.3777
34	.8766	.3289	.8649	.3418	.8527	.3544	.8401	.3665	.8272	.3781
36	.8762	.3293	.8645	.3423	.8523	.3548	.8397	.3669	.8267	.3785
38	.8758	.3298	.8641	.3427	.8519	.3552	.8393	.3673	.8263	.3789
40	.8754	.3302	.8637	.3431	.8515	.3556	.8389	.3677	.8258	.3793
42	.8751	.3307	.8633	.3435	.8511	.3560	.8384	.3680	.8254	.3796
44	.8747	.3311	.8629	.3440	.8507	.3564	.8380	.3684	.8249	.3800
46	.8743	.3315	.8625	.3444	.8502	.3568	.8376	.3688	.8245	.3804
48	.8739	.3320	.8621	.3448	.8498	.3572	.8372	.3692	.8241	.3808
50	.8735	.3324	.8617	.3452	.8494	.3576	.8367	.3696	.8236	.3811
52	.8731	.3328	.8613	.3457	.8490	.3580	.8363	.3700	.8232	.3815
54	.8727	.3333	.8609	.3461	.8486	.3585	.8359	.3704	.8227	.3819
56	.8724	.3337	.8605	.3465	.8482	.3589	.8354	.3708	.8223	.3823
58	.8720	.3341	.8601	.3469	.8477	.3593	.8350	.3712	.8218	.3826
60	.8716	.3346	.8597	.3473	.8473	.3597	.8346	.3716	.8214	.3830
$c+f$										
0.40	0.38	0.14	0.37	0.15	0.37	0.15	0.37	0.16	0.36	0.16
0.60	0.56	0.21	0.56	0.22	0.55	0.23	0.55	0.24	0.55	0.24
0.75	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30	0.68	0.31
1.00	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40	0.91	0.41
1.25	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50	1.14	0.52

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

$H = \cos^2 v$ and $\frac{1}{2} \sin 2v = V$

Min.	25°		26°		27°		28°		29°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.8214	.3830	.8078	.3940	.7939	.4045	.7796	.4145	.7650	.4240
2	.8209	.3834	.8074	.3944	.7934	.4049	.7791	.4148	.7645	.4243
4	.8205	.3838	.8069	.3947	.7930	.4052	.7786	.4152	.7640	.4246
6	.8201	.3841	.8065	.3951	.7925	.4055	.7781	.4155	.7635	.4249
8	.8196	.3845	.8060	.3954	.7920	.4059	.7777	.4158	.7630	.4253
10	.8192	.3849	.8055	.3958	.7915	.4062	.7772	.4161	.7625	.4256
12	.8187	.3853	.8051	.3961	.7911	.4066	.7767	.4165	.7620	.4259
14	.8183	.3856	.8046	.3965	.7906	.4069	.7762	.4168	.7615	.4262
16	.8178	.3860	.8041	.3969	.7901	.4072	.7757	.4171	.7610	.4265
18	.8174	.3864	.8037	.3972	.7896	.4076	.7752	.4174	.7605	.4268
20	.8169	.3867	.8032	.3976	.7892	.4079	.7748	.4177	.7600	.4271
22	.8165	.3871	.8028	.3979	.7887	.4082	.7742	.4181	.7595	.4274
24	.8160	.3875	.8023	.3983	.7882	.4086	.7738	.4184	.7590	.4277
26	.8156	.3878	.8018	.3986	.7877	.4089	.7733	.4187	.7585	.4280
28	.8151	.3882	.8014	.3990	.7873	.4092	.7728	.4190	.7580	.4283
30	.8147	.3886	.8009	.3993	.7868	.4096	.7723	.4193	.7575	.4286
32	.8142	.3889	.8004	.3997	.7863	.4099	.7718	.4197	.7570	.4289
34	.8138	.3893	.8000	.4000	.7858	.4102	.7713	.4200	.7565	.4292
36	.8133	.3897	.7995	.4004	.7854	.4106	.7709	.4203	.7560	.4295
38	.8128	.3900	.7990	.4007	.7849	.4109	.7704	.4206	.7555	.4298
40	.8124	.3904	.7986	.4011	.7844	.4112	.7699	.4209	.7550	.4301
42	.8119	.3908	.7981	.4014	.7839	.4116	.7694	.4212	.7545	.4304
44	.8115	.3911	.7976	.4018	.7834	.4119	.7689	.4215	.7540	.4307
46	.8110	.3915	.7972	.4021	.7830	.4122	.7684	.4219	.7535	.4310
48	.8106	.3918	.7967	.4024	.7825	.4126	.7679	.4222	.7530	.4313
50	.8101	.3922	.7962	.4028	.7820	.4129	.7674	.4225	.7525	.4316
52	.8097	.3926	.7958	.4031	.7815	.4132	.7669	.4228	.7520	.4318
54	.8092	.3929	.7953	.4035	.7810	.4135	.7664	.4231	.7515	.4321
56	.8087	.3933	.7948	.4038	.7806	.4139	.7659	.4234	.7510	.4324
58	.8083	.3936	.7944	.4042	.7801	.4142	.7655	.4237	.7505	.4327
60	.8078	.3940	.7939	.4045	.7796	.4145	.7650	.4240	.7500	.4330
$c+f$										
0.40	0.36	0.17	0.36	0.18	0.36	0.18	0.35	0.19	0.35	0.19
0.60	0.54	0.26	0.53	0.27	0.53	0.28	0.53	0.28	0.52	0.29
0.75	0.68	0.32	0.67	0.33	0.66	0.35	0.66	0.36	0.65	0.36
1.00	0.90	0.43	0.89	0.45	0.89	0.46	0.88	0.47	0.87	0.48
1.25	1.13	0.54	1.12	0.56	1.11	0.58	1.10	0.59	1.09	0.60

$(c+f) \cos v$ and $(c+f) \sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

H = Cos²v and ½sin 2v. = V

Min.	30°		31°		32°		33°		34°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.7500	.4330	.7347	.4415	.7192	.4494	.7034	.4568	.6873	.4636
2	.7495	.4333	.7342	.4417	.7187	.4497	.7028	.4570	.6868	.4638
4	.7490	.4336	.7337	.4420	.7181	.4499	.7023	.4572	.6862	.4640
6	.7485	.4339	.7332	.4423	.7176	.4502	.7018	.4575	.6857	.4642
8	.7480	.4342	.7327	.4426	.7171	.4504	.7012	.4577	.6851	.4645
10	.7475	.4345	.7322	.4428	.7166	.4507	.7007	.4579	.6846	.4647
12	.7470	.4347	.7316	.4431	.7160	.4509	.7002	.4582	.6841	.4649
14	.7465	.4350	.7311	.4434	.7155	.4512	.6996	.4584	.6835	.4651
16	.7460	.4353	.7306	.4436	.7150	.4514	.6991	.4586	.6830	.4653
18	.7455	.4356	.7301	.4439	.7145	.4517	.6986	.4589	.6824	.4655
20	.7449	.4359	.7296	.4442	.7139	.4519	.6980	.4591	.6819	.4657
22	.7444	.4362	.7291	.4444	.7134	.4522	.6975	.4593	.6814	.4660
24	.7439	.4365	.7285	.4447	.7129	.4524	.6970	.4596	.6808	.4662
26	.7434	.4367	.7280	.4450	.7124	.4527	.6964	.4598	.6803	.4664
28	.7429	.4370	.7275	.4452	.7118	.4529	.6959	.4600	.6797	.4666
30	.7424	.4373	.7270	.4455	.7113	.4532	.6954	.4603	.6792	.4668
32	.7419	.4376	.7265	.4458	.7108	.4534	.6948	.4605	.6786	.4670
34	.7414	.4379	.7260	.4460	.7103	.4536	.6943	.4607	.6781	.4672
36	.7409	.4382	.7254	.4463	.7097	.4539	.6938	.4609	.6776	.4674
38	.7404	.4384	.7249	.4466	.7092	.4541	.6932	.4612	.6770	.4676
40	.7399	.4387	.7244	.4468	.7087	.4544	.6927	.4614	.6765	.4678
42	.7393	.4390	.7239	.4471	.7081	.4546	.6921	.4616	.6759	.4680
44	.7388	.4393	.7234	.4473	.7076	.4549	.6916	.4618	.6754	.4682
46	.7383	.4395	.7228	.4476	.7071	.4551	.6911	.4621	.6748	.4684
48	.7378	.4398	.7223	.4479	.7066	.4553	.6905	.4623	.6743	.4686
50	.7373	.4401	.7218	.4481	.7060	.4556	.6900	.4625	.6737	.4688
52	.7368	.4404	.7213	.4484	.7055	.4558	.6895	.4627	.6732	.4690
54	.7363	.4407	.7208	.4486	.7050	.4561	.6889	.4629	.6726	.4692
56	.7358	.4409	.7202	.4489	.7044	.4563	.6884	.4632	.6721	.4694
58	.7352	.4412	.7197	.4491	.7039	.4565	.6878	.4634	.6716	.4696
60	.7347	.4415	.7192	.4494	.7034	.4568	.6873	.4636	.6710	.4698
$c+f$										
0.40	0.35	0.19	0.34	0.21	0.34	0.21	0.33	0.22	0.33	0.22
0.60	0.53	0.29	0.52	0.31	0.51	0.32	0.50	0.32	0.50	0.33
0.75	0.66	0.36	0.64	0.39	0.64	0.40	0.63	0.41	0.62	0.42
1.00	0.88	0.48	0.86	0.52	0.85	0.53	0.84	0.54	0.83	0.56
1.25	1.10	0.60	1.07	0.65	1.06	0.66	1.05	0.67	1.04	0.70

$(c+f)$ cos v and $(c+f)$ sin v.

Natural functions.

Stadia Coefficients, Vertical Rod.

$H = \text{Cos}^2 v$ and $\frac{1}{2}\sin 2v = V$

Min.	35°		36°		37°		38°		39°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.6710	.4698	.6545	.4755	.6378	.4806	.6210	.4852	.6040	.4891
2	.6705	.4700	.6539	.4757	.6373	.4808	.6204	.4853	.6034	.4892
4	.6699	.4702	.6534	.4759	.6367	.4810	.6198	.4854	.6028	.4893
6	.6694	.4704	.6528	.4761	.6361	.4811	.6193	.4856	.6023	.4894
8	.6688	.4706	.6523	.4762	.6356	.4813	.6187	.4857	.6017	.4896
10	.6683	.4708	.6517	.4764	.6350	.4814	.6181	.4858	.6011	.4897
12	.6677	.4710	.6512	.4766	.6345	.4816	.6176	.4860	.6005	.4898
14	.6672	.4712	.6506	.4768	.6339	.4817	.6170	.4861	.6000	.4899
16	.6666	.4714	.6501	.4769	.6333	.4819	.6164	.4863	.5994	.4900
18	.6661	.4716	.6495	.4771	.6328	.4820	.6159	.4864	.5988	.4901
20	.6655	.4718	.6490	.4773	.6322	.4822	.6153	.4865	.5983	.4902
22	.6650	.4720	.6484	.4775	.6317	.4824	.6147	.4867	.5977	.4904
24	.6644	.4722	.6478	.4776	.6311	.4825	.6142	.4868	.5971	.4905
26	.6639	.4724	.6473	.4778	.6305	.4827	.6136	.4869	.5965	.4906
28	.6633	.4726	.6457	.4780	.6300	.4828	.6130	.4871	.5960	.4907
30	.6628	.4728	.6462	.4782	.6294	.4830	.6125	.4872	.5954	.4908
32	.6622	.4729	.6456	.4783	.6288	.4831	.6119	.4873	.5948	.4909
34	.6617	.4731	.6451	.4785	.6283	.4833	.6113	.4874	.5943	.4910
36	.6611	.4733	.6445	.4787	.6277	.4834	.6108	.4876	.5937	.4911
38	.6606	.4735	.6440	.4788	.6272	.4836	.6102	.4877	.5931	.4912
40	.6600	.4737	.6434	.4790	.6266	.4837	.6096	.4878	.5925	.4914
42	.6595	.4739	.6429	.4792	.6260	.4839	.6091	.4880	.5920	.4915
44	.6589	.4741	.6423	.4793	.6255	.4840	.6085	.4881	.5914	.4916
46	.6584	.4743	.6417	.4795	.6249	.4841	.6079	.4882	.5908	.4917
48	.6578	.4744	.6412	.4797	.6244	.4843	.6074	.4883	.5903	.4918
50	.6573	.4746	.6406	.4798	.6238	.4844	.6068	.4885	.5897	.4919
52	.6567	.4748	.6400	.4800	.6232	.4846	.6062	.4886	.5891	.4920
54	.6562	.4750	.6395	.4801	.6226	.4847	.6057	.4887	.5885	.4921
56	.6556	.4752	.6389	.4803	.6221	.4849	.6051	.4888	.5880	.4922
58	.6551	.4753	.6384	.4805	.6215	.4850	.6045	.4890	.5874	.4923
60	.6545	.4755	.6378	.4806	.6210	.4852	.6040	.4891	.5868	.4924
$c+f$										
0.40	0.33	0.23	0.32	0.24	0.32	0.24	0.31	0.25	0.31	0.25
0.60	0.49	0.34	0.49	0.35	0.48	0.36	0.47	0.37	0.46	0.38
0.75	0.61	0.43	0.61	0.44	0.60	0.45	0.59	0.46	0.58	0.47
1.00	0.82	0.57	0.81	0.59	0.80	0.60	0.79	0.62	0.78	0.63
1.25	1.02	0.71	1.01	0.74	1.00	0.75	0.98	0.78	0.97	0.79

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Stadia Coefficients, Vertical Rod.

$H = \cos^2 v$ and $\frac{1}{2}\sin 2v. = V$

Min.	40°		41°		42°		43°		44°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0	.5868	.4924	.5696	.4951	.5523	.4973	.5349	.4988	.5174	.4997
2	.5863	.4925	.5690	.4952	.5517	.4973	.5343	.4988	.5169	.4997
4	.5857	.4926	.5684	.4953	.5511	.4974	.5337	.4989	.5163	.4997
6	.5851	.4927	.5679	.4954	.5505	.4974	.5331	.4989	.5157	.4998
8	.5845	.4928	.5673	.4955	.5500	.4975	.5325	.4989	.5151	.4998
10	.5840	.4929	.5667	.4955	.5494	.4976	.5320	.4990	.5145	.4998
12	.5834	.4930	.5661	.4956	.5488	.4976	.5314	.4990	.5140	.4998
14	.5828	.4931	.5656	.4957	.5482	.4977	.5308	.4990	.5134	.4998
16	.5822	.4932	.5650	.4958	.5476	.4977	.5302	.4991	.5128	.4998
18	.5817	.4933	.5644	.4958	.5471	.4978	.5296	.4991	.5122	.4999
20	.5811	.4934	.5638	.4959	.5465	.4978	.5291	.4992	.5116	.4999
22	.5805	.4935	.5633	.4960	.5459	.4979	.5285	.4992	.5110	.4999
24	.5799	.4936	.5627	.4961	.5453	.4979	.5279	.4992	.5105	.4999
26	.5794	.4937	.5621	.4961	.5447	.4980	.5273	.4993	.5099	.4999
28	.5788	.4938	.5615	.4962	.5442	.4980	.5267	.4993	.5093	.4999
30	.5782	.4938	.5609	.4963	.5436	.4981	.5262	.4993	.5087	.4999
32	.5776	.4939	.5604	.4963	.5430	.4981	.5256	.4993	.5081	.4999
34	.5771	.4940	.5598	.4964	.5424	.4982	.5250	.4994	.5076	.4999
36	.5765	.4941	.5592	.4965	.5418	.4982	.5244	.4994	.5070	.5000
38	.5759	.4942	.5586	.4966	.5413	.4983	.5238	.4994	.5064	.5000
40	.5753	.4943	.5581	.4966	.5407	.4983	.5233	.4995	.5058	.5000
42	.5748	.4944	.5575	.4967	.5401	.4984	.5227	.4995	.5052	.5000
44	.5742	.4945	.5569	.4968	.5395	.4984	.5221	.4995	.5047	.5000
46	.5736	.4946	.5563	.4968	.5389	.4985	.5215	.4995	.5041	.5000
48	.5730	.4946	.5557	.4969	.5384	.4985	.5209	.4996	.5035	.5000
50	.5725	.4947	.5552	.4969	.5378	.4986	.5204	.4996	.5029	.5000
52	.5719	.4948	.5546	.4970	.5372	.4986	.5198	.4996	.5023	.5000
54	.5713	.4949	.5540	.4971	.5366	.4987	.5192	.4996	.5017	.5000
56	.5707	.4950	.5534	.4971	.5360	.4987	.5186	.4997	.5012	.5000
58	.5702	.4951	.5528	.4972	.5355	.4987	.5180	.4997	.5006	.5000
60	.5696	.4951	.5523	.4973	.5349	.4988	.5174	.4997	.5000	.5000
$c+f$										
0.40	0.31	0.25	0.30	0.26	0.30	0.27	0.29	0.27	0.29	0.28
0.60	0.46	0.38	0.45	0.39	0.44	0.40	0.44	0.41	0.43	0.41
0.75	0.57	0.48	0.57	0.49	0.55	0.50	0.55	0.51	0.54	0.52
1.00	0.77	0.64	0.75	0.66	0.74	0.67	0.73	0.68	0.72	0.68
1.25	0.96	0.80	0.94	0.82	0.92	0.84	0.91	0.85	0.90	0.86

$(c+f)\cos v$ and $(c+f)\sin v$.

Natural functions.

Money Exchange

From a Report of the Director of the Mint, 1913, and from the Sec. of Treas. July 1, 1914

Country	Unit		Equivalent
Abyssinia	Menelik	= 0.4604	
Afghanistan	Tilla	= 2.8334	
Argentine	Peso	= 0.9647	= 100 centavos
	Argentine	= 4.8236	
Austro-Hungary	Crown	= 0.2026	= 100 hellers = ½ florin
	Ducat	= 2.2877	
Belgium	Franc	= 0.1929	= 100 centimes
Bolivia	Boliviano	= 0.3893	= 100 centavos
Brazil	Milreis	= 0.5463	= 1000 reis
Br. Honduras	Dollar	= 1.00	
Bulgaria	Leva	= 0.1929	= 1 lew
Canada	Dollar	= 1.00	= 100 cents
Chili	Peso	= 0.365	= 100 centavos
	Condor	= 7.2995	= 20 pesos = 4 escudo
China	Tael	= 0.63 to 0.70	= 100 fun = 1000 cash
	Dollar'	= 0.504	= 1 yuan
Columbia	Dollar	= 1.00	= 1 peso
Costa Rica	Colon	= 0.4653	= 100 centimos
Cuba (No National Standard)		=	Sp. & Amer. money used
Denmark	Crown	= 0.268	= 100 öre
Equador	Sucre	= 0.4866	= 100 cents = 5 peseta = 10 reals
Egypt	Pound	= 4.943	= 100 piasters
England (British Empire)	Pound	= 4.8665	= 20 shillings
	Crown	= 1.2166	= 60 pence
	Florin	= 0.4866	= 96 farthings
Finland	Markka	= 0.193	= 100 pennia
France, Indo China, Tunis, etc.	Franc	= 0.1929	= 100 centimes
	Napoleon	= 1.9294	= 10 francs = 200 sous
German Empire	Mark	= 0.2381	= 100 pfennigs
	Crown	= 2.382	= 10 marks
Greece and Crete	Drachma	= 0.1929	= 100 lepta
Guatemala	Peso, gold	= 0.9642	Silver peso = 0.422
Haiti	Gourde	= 0.9647	= 200 centimes
Honduras	Peso	= 0.422	= 8 reales
Honkong	Dollar	= 0.431	
India and Ceylon	Rupee	= 0.3244	= 16 anna = 64 pice
Italy	Lira	= 0.1929	= 100 centesimi
Japan, Korea and Formosa	Yen	= 0.4984	= 100 sen = 1000 rin
Liberia	Dollar	= 1.00	
Mexico	Peso	= 0.4984	= 100 centavos
Montenegro	Perper	= 0.2026	= 1 Austrian crown
Morocco	Rial	= 0.4446	= 10 ounces
Netherlands	Florin	= 0.40195	= 100 cents
	Ducat	= 2.2826	
Newfoundland	Dollar	= 1.014	
Nicaragua	Cordova	= 1.00	= 100 centavos
Norway	Crown	= 0.268	= 100 öres
Panama	Balboa	= 1.00	= 2 pesos
Paraguay	Peso	= 0.422	= 100 cents
Persia	Tomani	= 1.7207	= 10 krans
Peru	Quintos	= 0.9733	= ⅕ libra = 2 sol = 20 dineros
Philippines	Peso	= 0.50	
Portugal	Milreis, G	= 1.08	= 1 escudo (S.)
Roumania	Lei, gold	= 0.193	= 1 leu (S.) = 100 bani
Russia	Ruble	= 0.5145	= 100 kopecks
Salvador	Peso	= 0.422	= 8 reales
San Domingo	Dollar	= 1.00	
Servia	Dinar	= 0.1929	= 100 paras
Siam	Tical	= 0.3708	= 4 salungs
Spain and Andorra	Piesta	= 0.9129	= 100 centimos
Straits Settlements	Dollar	= 0.5677	
Sweden	Crown	= 0.268	= 100 öre
Switzerland	Franc	= 0.1929	= 100 centimes
Turkey	Lira	= 4.3966	= 100 piasters
Uruguay	Peso	= 1.034	= 100 centimos
Venezuela	Bolivar	= 0.1929	

Index

Bausch & Lomb Observatory

Equipped with 27 cm Equatorial, Electric Drive for Observation Floor and Dome, also Complete Time Service including 3-in. Transit, Sidereal and Mean Time Clocks, Chronograph, etc.

80 cm Coincidence Range Finder for Infantry and Artillery.

Note:—The cut at top of opposite page shows instrument mounted on short tripod for prone observations.

THE ONLY INSTRUMENTS of note that have been proposed to supplement the transit, level and alidade for topographical surveys are the photo-theodolite which, in expert hands, is more accurate and more thorough than the plane table and the telemeter which provides the most rapid known method of obtaining approximate distances.

For military work the telemeter is known as the Range Finder. There are various types but generally the basic optical principles are alike. The instrument contains an optical base-line with suitable prisms to project two images of the field upon a coincidence plane in the focus of the eyepiece. The amount of deflection required to perfect a continuous and symmetrical image of the object sighted, depends upon its distance from the instrument.

Naturally the longer the optical base the greater the possibilities for accurate and consistent observations. Those built for the Navy or Coast Defence and designed for permanent mounting attain the length of 30 ft. and over and reduce errors to less than one percent. Where portability is the first consideration, as for Infantry use, the instrument is reduced in length to 80 cm and in weight to about 5 kg. In this type the effective range lies between 400 and 5000 yds. which is sufficient for the required purpose.

Range Finders for the shorter distances encountered in topography would have to be constructed in a still shorter base with increasing probabilities of error that doubtless would exceed those now attributable to stadia surveys. The larger models have self contained triple mirror adjusting apparatus, astigmatizer for observing luminous spots at night, sun light filters and variable magnification. The system of collimation must be very exact. It has been found advantageous to mount the optical system, with objective penta-prisms, on wires under high tension; not only to preserve perfect alignment but to overcome flexure and add stiffness to the entire structure.